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# The Philosophy of Re-entry by Spencer-Brown

Lecture for the formlabor in Berlin on January 31 2020 (substantially extended version)

## Introduction

The doctrine of re-entry is at the heart of the *Laws of Form*.

Re-entry is not ...

– resonance, feedback, or a response from another person or an external observer

– a reaction to an external influence (e.g., a cybernetic thermostat in response to heat input)

– reflection off an external mirror

but rather...

– that something detaches itself from a given thing and its form in a formally describable process,

– creates a virtual space for its movement,

– transforms itself there through its own momentum,

– possesses self-reflexivity (an internal observer of itself),

– and ultimately returns to the original set as something new,

– thereby setting it into an inner vibration and transforming it,

– until the element that has returned, altered by its re-entry, is sutured.

This process traverses a virtual space and is therefore comprehensible **only through language**, which it generates from within itself. It follows fixed, recurring rules and is **algorithmic**. With this approach, Spencer-Brown finds an answer to Bertrand Russell's antinomy of negative self-reference and Ludwig Wittgenstein's theory of language, who were his role models at Cambridge.

He shows how language and algorithms develop their autonomy and momentum through re-entry. Language is not merely a mapping of things onto words, but is based on the imagination of those who speak to one another—their ability to form new words in their minds and to use these words to

create new realities. Imagination, in turn, is not based on mere intuition; rather, it is through intuition that the associations between words—already inherent in the words themselves—become apparent. This is a unified process that cannot be reduced to any one of its constituent elements.

Since the modern era, this way of thinking has been regarded as mystical, magical, or romantic—as the Romantics were the only ones to oppose the Enlightenment. But the original creation myths inevitably took the form of a narrative showing how the world emerged from words. Even mathematics, in its early stages, could not help but understand the origin of its operands and operators (numbers, geometric elements, and methods of construction and calculation) as stemming from a primordial myth or within a realm of ideas (Platonism). With the concept of “re-entry,” Spencer-Brown demonstrates how language and algorithms emerge in a virtual space that arises from the world of being. This approach also offers a pathway to understanding artificial intelligence, particularly when logical machines—composed of algorithms and neural networks—are believed to be capable of arriving at new results on their own and transforming the world. Artificial intelligence creates virtual worlds that return via "re-entry." This can generate both fascination and fear. Spencer-Brown's theory of "re-entry" offers a way to understand what happens and how it happens.

**Outline**: In two steps, we first provide a technical description of what Spencer-Brown means by "re-entry": What distinguishes re-entry from iteration, recursion, and backtracking, and how is re-entry formally derived from primary algebra (echelon, Consequence C7)? – The embedding of re-entry into the *Laws of Form* and the extension of re-entry to tunnels, time, memory, and the modulator are discussed in the parallel article on the *Laws of Form*.

The remaining sections are devoted to the philosophical interpretation of re-entry: Re-entry describes something sacred (Kunze); it allows for a distinction between positive and negative language (Gotthard Günther); and if it fails, it can lead to existential crises (Kristeva).

# Iteration, Rekursion, Backtracking, Re-entry

What distinguishes iteration, recursion, reset (backtracking) and re-entry?

Iteration is the multiple repetition of the same calculation process, in programming a loop. This means that each new step in the calculation takes the result of the previous step and continues according to a consistent rule. The first counting step resulted in 2. Counting continues from 2, and so on.

| | |
|---|---|
| Initial number | 1 |
| Loop with fixed step increment 1 | 1 + 1 = 2<br>2 + 1 = 3<br>3 + 1 = 4<br>… |

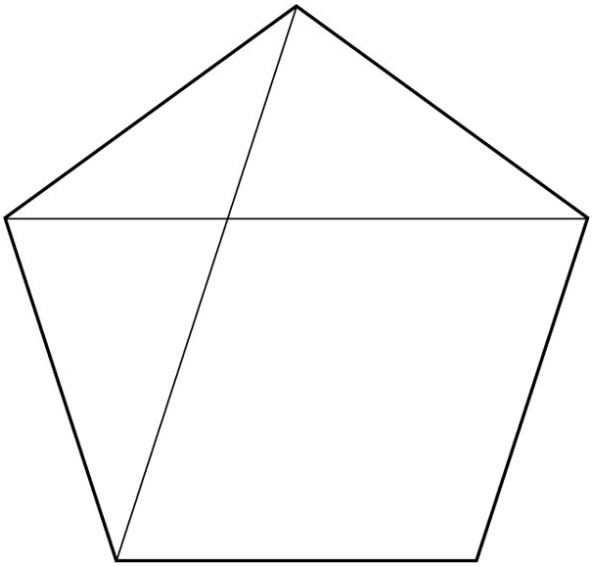
Counting is also a recursion: This means that each new calculation step takes the result of the previous calculation step as its starting point. The first counting step has led to the 2. Counting continues from the 2 and so on.

The recursion need not be limited to the last result. In the Fibonacci sequence, the sum of the last two elements of the sequence results in the new sequence element. The ratio of two consecutive numbers changes continuously and approaches the golden ratio Φ, which—unlike the calculated numbers—is not an irrational number:

| | | | |
|---|---|---|---|
| Initial numbers | 0<br>1 | | |
| Loop with variable step increment | 0 + 1 = 1 | | |
| | 1 + 1 = 2 | 2 / 1 | = 2 |
| | 2 + 1 = 3 | 3 / 2 | = 1,5 |
| | 3 + 2 = 5 | 5 / 3 | = 1,66… |
| | 5 + 3 = 8 | 8 / 5 | = 1,6 |
| | 8 + 5 = 13 | 13 / 8 | = 1,625 |
| | … | | → 1,618… (Φ, Golden Ratio) |

Johannes Kepler had already noticed this; see various references in his *Harmonice Mundi*, published in 1619. The ratio of two consecutive terms in the Fibonacci sequence, ($a_{n+1} : a_n$), approaches the Golden Ratio without ever reaching it. Since the Golden Ratio is the irrational number $(1 + \sqrt{5}) / 2$, it is given its own symbolic name: Φ. In this way, a sequence of natural numbers is used to approximate a number that is not natural and must be extended (adjoined) to the set of all natural numbers through a re-entry. Since it is a ratio value within the series, it can be called an *eigenvalue* with some justification. From then on, it has been regarded as the inner harmony of growth as well as of beauty.

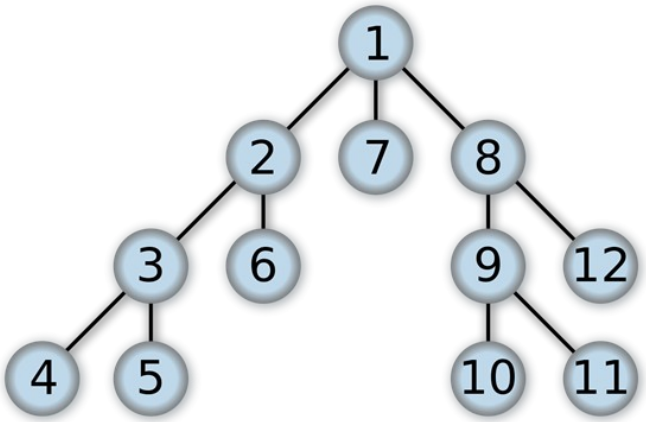
Where is the golden ratio located, if it is not a natural number? Since Pythagoras it is known that in the pentagon the diagonals intersect in such a way that the two halves of a diagonal are in proportion to the golden ratio:



Another example: If you want to calculate the square root of 2 by hand without a calculator, you also use a recursive approach by calculating one decimal place at a time:

| | |
|---|---|
| 1st digit: | $1 \cdot 1 = 1$<br>$2 \cdot 2 = 4$<br>4 is too large, therefore the 1st digit is 1 |
| 2nd digit: | $1{,}5 \cdot 1{,}5 = 2{,}25$<br>$1{,}4 \cdot 1{,}4 = 1{,}96$<br>2,25 is too large, therefore the 2nd digit is 4 |
| 3rd digit: | $1{,}41 \cdot 1{,}41 = 1{,}9881$<br>$1{,}42 \cdot 1{,}42 = 2{,}0164$<br>2,0164 is too large, therefore the 3rd digit is 1 |
| … | |

Backtracking (reset procedure). The elementary example is finding one's way through a labyrinth.




**Deep search of Backtracking**

Author: Alexander Drichel - Eigenes Werk, CC BY-SA 3.0, Link

– The first step is to branch to the left at every fork until a dead end is reached.
– The last step before the dead end is marked as a wrong way. From this fork, the next possible path to the left is taken.
– If all paths from a fork have been marked as wrong paths, one goes back one step to the previous fork and searches for a path from there.
– Once the exit is found, the procedure stops.

The backtracking includes iterations and recursions: Individual steps, similar to each other, are executed, and after each dead end, the procedure is recursively recalled. However, backtracking contains more than just the simplest recursion, it also contains the marking of wrong paths. – Practical applications of backtracking are the Knight's tour, »to find a route for a knight on an empty chessboard, on which he visits each square exactly once«, the route search, various game programs or the programming language PROLOG.

What is the difference between the **Re-entry** and these algorithmic procedures? Wherever the three dots ›…‹ were previously entered, the three dots are replaced by a new number that was not contained in the previous number space and is taken up there by a re-entry. Mathematicians have always done this – without being aware of it – by introducing new number symbols and adding them to the original set of numbers. The default case are irrational numbers like the $\sqrt{2}$, for which the new symbol $\sqrt{\ }$ is introduced, as well as the golden ratio Φ, which has already been mentioned.

# The echelon without limit

**(1) Initial question**. How is it possible, within the *Laws of Form* and using only the tools of primary arithmetic and primary algebra—that is, using only the single symbol, the ⌐, which serves equally as operand of the *marked* and as operator of the *marking*—to describe an infinite progression such as that found in the examples of counting and the Fibonacci sequence? Spencer-Brown goes even further: He does not want to choose counting as an example, but rather, conversely, to construct a sequence from which counting—and, by extension, concepts such as time, memory, and modulation—can be *logically derived*. Therefore, he does not choose a sequence in which his basic symbol, ⌐, is arranged consecutively side by side or nested within one another. So he does not choose a sequence such as:

⌐, ⌐ ⌐, ⌐ ⌐ ⌐ …, or ⌐, ⌐‖, ⌐‖‖, ….

**(2) Spencer-Brown's answer and his hidden programme**. He searches for a form that *multiplies itself*. If this succeeds, this form can call itself infinitely often. This leads to the desired result: replacing the three dots '…' in this sequence with a new symbol. Spencer-Brown chooses the letter *f* as the new symbol.

He leaves open why he chooses the letter *f*. Here, in my interpretation, he follows a hidden programme, which I want to lay open from the outset in order to bring out the inner orientation of the presentation of re-entry. By choosing the letter *a* for the sequence of repeating symbols and *f* for the newly emerging symbol, he draws on Frege, for whom *a* stands for argument and *f* for function. His aim is to derive, from arguments *a*, the overarching function *f* that is defined over these arguments. This is nothing less than the genesis of the function *f* out of its own arguments *a*, without *f* being reducible to *a*; and he completes it with the modulator function, which likewise converts an input *a* into an output *f* (LoF, 66f.).

**(3) The hidden tradition**. Scheier has succinctly summarized Frege's position: While Frege speaks of functions f(a) and clearly distinguishes the function *f( )*—with its empty placeholder—from the argument *a*, with which the function is only subsequently to be *saturated*, Scheier recognizes in this an ontological distinction that is implicit in Frege's work and proposes a novel formalization to enable operations on it: ›f | a‹, “where the symbol ›|‹ denotes the difference that, while it can be shifted by supplementary functions, can no longer be resolved in the manner of Hegel” (Scheier, 18, my translation). He explains in a note: “The vertical line indicates the onto-logical difference and is not identical with the logical Sheffer bar” (Scheier, 134, note 36, my translation).

The “functional constitution of being” is substantiated with quotations from Wittgenstein's *Tractatus* and summarized:

> “'Like Frege and Russell I construe a proposition as a function of the expressions contained in it.' (TLP 3.318) 'To give the essence of a proposition means to give the essence of all description, and thus the essence of the world.' (TLP 5.4711). The function reflects the logical space (TLP 1.13), the argument the substance (TLP 2.021). The world, reflected 'sub specie aeterni' as a saturated function, is viewed therein 'a whole—a limited whole' (TLP 6.45). The difference '|', which constantly accompanies the chaining and hierarchization of propositions, cannot be expressed either as a function or as an argument, but is (as here) supplemented by both—it is *felt* 'as such' (ibid.)” (Scheier, 21) – “The operation is not its result: f | a.” (Scheier, 67)

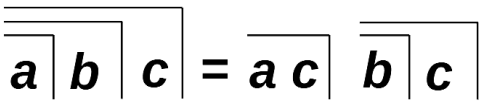
**(4) The open question.** The onto-logical difference cannot be sublated. But it can be handled with the operation introduced by Scheier, which remains within language and does not remove the difference but works (operates) in and with it. It is precisely at this point that Spencer-Brown wants to go beyond Frege, Russell, and Wittgenstein, and to show how, with re-entry, there can be an operation that leads from *a* to *f*. In this way he resolves the self-reference “forbidden” by Russell (in re-entry, *f* arises from its own arguments *a*), by introducing—in a strictly logical calculus—oscillation, time, and indeterminacy, with which a logical presentation of the negatively self-

referential liar paradox ›this proposition is false‹ and the antinomies developed from it succeeds. And at the same time he has an answer to Wittgenstein's concern that language should *show itself in itself*, where the function stands for the elementary building block of language (a proposition) and the argument for the names used by language, which mean their objects (TLP 3.203). To be precise: in the Tractatus, the name is not simply an argument, but that which occupies the argument place. Wittgenstein had come upon the open question: “What expresses *itself* in language, we cannot express by means of language.” (TLP 4.121) Wittgenstein held it to be impossible for language to catch up, through an operation that runs within language itself, with the form that shows itself in it. Spencer-Brown gives, with re-entry, an answer that was unthinkable for Wittgenstein. He remains faithful to him at the same time, insofar as re-entry too does not transcend language: the form enters into its own interior without stepping out of it. The path there requires a state of its own, which Spencer-Brown described as creative over against the parasitic state of mere rule-following (LoF, 102) (parasitic in the sense of 'rule-dependent, incapable of forming its own distinctions', not in the biological or political sense; cf. Greek parásitos: “one who has his food at another's table”).

**(5) Outline.** In accordance with this far-reaching concern of Spencer-Brown, the argument proceeds in two steps. First, it is to be retraced how Spencer-Brown selects a particular form and shows re-entry upon it. In a second step, with Kunze, the question is to be asked how this leap from the arguments to the function can succeed. It must be an inference from the arguments to the function, for which there must be a middle term (terminus medius) capable of mediating this leap. With Spencer-Brown and with examples from the history of art and of the city, Kunze wishes to show that this can succeed only if the middle term oscillates and is hallowed. If, since the modern era, thinking of this kind has been accused of mysticism and magic, it is to be shown with Kunze in what way a new understanding of myth and of the sacred can be introduced.

## – The Echelon Without Limit

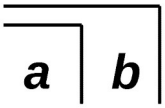
The *Consequence 7. Echelon* (LoF, 34), introduced and demonstrated (proved) in the *Primary Algebra*, is the sought form that multiplies itself. The term *echelon* comes from military usage and describes a high formation, the echelon formation. Each unit is positioned slightly offset behind the one in front, either to the right (right echelon) or to the left (left echelon). The word was adopted from the French *échelon* (rung of a ladder). It is therefore also translated as 'Staffelung' ('staggering') or 'Staffelstellung' ('staggered arrangement') in the German translation supervised by Spencer-Brown. In Spencer-Brown the form reads:



Spencer-Brown designates it, read from left to right, as *break*, and read from right to left, as *make*. An echelon is broken or made. But here it is initially less a question of how an echelon can be broken or made; rather, in this consequence its inner symmetry is recognized on the left side: *a*, *b*, and *c* are uniformly staggered like an echelon. From this arises the idea: how can such an echelon be multiplied step by step into the infinite? Can it be extended, as with Cantor, into an ladder that leads into the uncountable numbers?

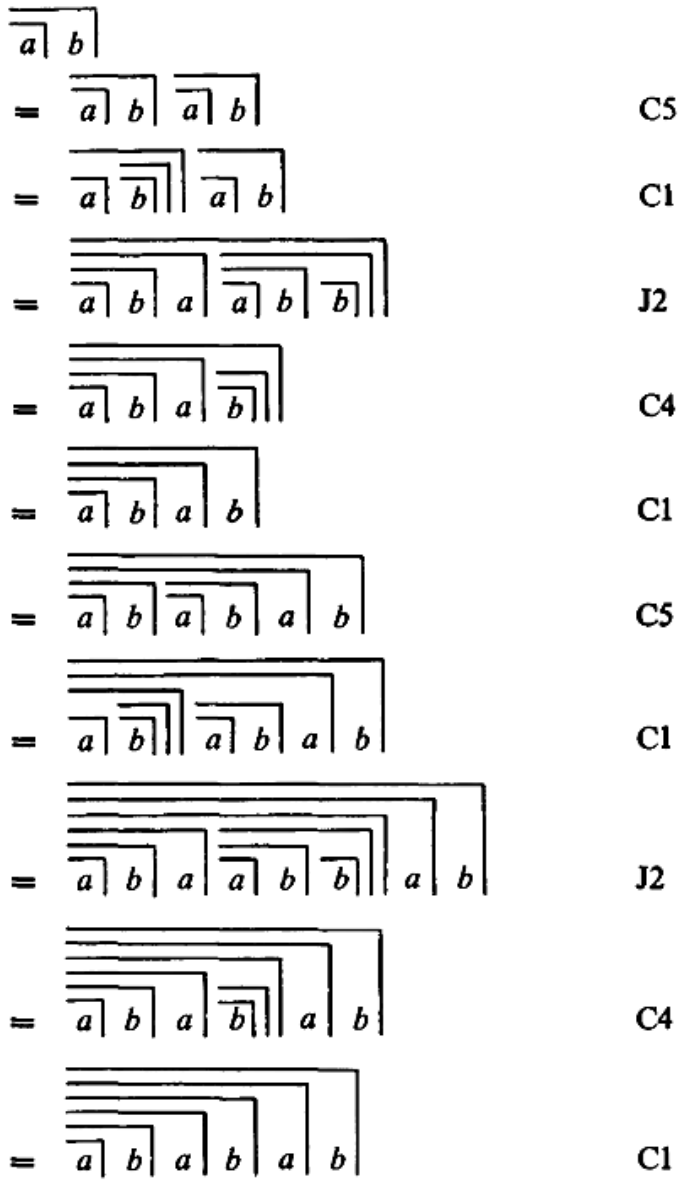
The echelon is simplified to its first two steps (rungs). This happens in Spencer-Brown by means of the *Ninth Canon. Rule of Demonstration* (LoF, 54). It states: "*A demonstration rests in a finite number of steps.*" (LoF, 54).

To explain this canon, Spencer-Brown goes back to the first two steps of the echelon (ibid.):

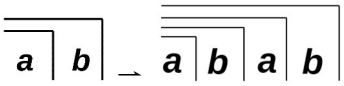



These two steps are to be continued, with increasing enlargement of the finite number, up to the limiting process of an infinite echelon.

To prove this formally turns out, surprisingly, to be substantially more complex than the demonstration of consequence 7 (which will later be discussed in greater depth with Kunze). Spencer-Brown shows how a sequence of 5 transformations can be applied to this simple sign, whereby the sign doubles after 5 steps, triples after a further 5 steps, and so on (LoF, 55). Spencer-Brown uses the rules C5, C1, J2, and C4, which had been introduced in his calculus. (Instead of rules he speaks of initials and consequences, in order to emphasize that the consequences can in turn be derived from the initials and subsequently stand ready as rules for operating in the primary algebra.) Recalculating this can be regarded as a demanding exercise in handling Spencer-Brown's calculus. For details see the appendix on Canon C9.



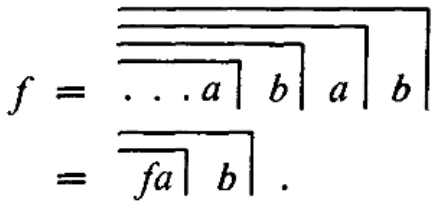

Or at a glance (LoF, 56):




With the symbol ›⎓‹ it is stated that both signs (forms) are of equal value. The same sequence of 5 individual steps can in turn be applied to the new sign (form), whereby it is lengthened once more. In this way a sequence of steps succeeds that corresponds to the successor relation and can be repeated infinitely often. Although Spencer-Brown introduced the echelon within the primary algebra as consequence C7 with three variables *a*, *b*, and *c* (LoF, 34), and not as here for two variables *a* and *b*, the result for him is “an echelon without limit” (LoF, 56). Or, in a slightly paraphrased formulation:

> “The next decisive step consists in imagining, *if we can* (LoF, 55), that the process of this step-sequence continues endlessly or timelessly. In space this would yield an echelon of a's and b's without limit. [...] Since the determination of the value of an expression is bound to the rule of finite demonstrability, no statement can be made about the value of an echelon *without limits*.” (Schönwalder-Kuntze et al., 175, my translation)

In the result, Spencer-Brown writes down a diagram with which the form is defined that is to be admitted into the primary algebra through re-entry:




**E1 Re-entry**
(LoF, 56)

Diagram E1 is to be read block by block from right to left, contrary to the usual reading direction. The formula E1, written down as a definition, emphasizes the recursion: the element respectively reached is designated as *f* and called anew in the following step. The transition from *b* to *a* is repeated arbitrarily often. The three dots are taken over from the corresponding processes known from ordinary arithmetic, where counting continues beyond ›n‹ and ›n + 1‹, or an irrational number is approximated arbitrarily closely; and with them he describes the form that emerges from the infinite process of the echelon.

The summary of the infinitely many crossings from *b* to *a* to *b* to *a*, and so on, in *f* is bound to three conditions:

(a) “a clearly determined sequence of steps, that is, a clear construction instruction”,
(b) it “is to be continued continuously”,
(c) and it contains a “pattern of self-sameness” (Schönwalder-Kuntze et al., 175, my translation).

> “The key is to see that the crossed part of the expression at every even depth is identical with the whole expression, which can thus be regarded as re-entering its own inner space at any even depth.” (LoF, 56)

In the result, *f* has a multiple meaning:

– *f* describes in E1 the place at which the recursion occurs, and replaces the three indeterminate dots ›…‹.

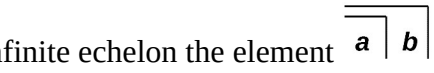
– If the recursions are counted along, then *f* is the name for the infinitely distant, last recursion. In this meaning, *f* is comparable to the golden ratio Φ or to the transfinite number ω. If *f* is understood as the name of the specific course-form of all recursions executed in E1, it is comparable to the eigenvalues considered by v. Foerster.

– Finally, *f* in E1 is the name of the form with which the infinitely often repeated recursion is described. E1 can be read as the self-referentially written definition of *f*. This form, bearing the name *f*, stands for the element that is to be admitted (adjoined) into the primary algebra through re-entry.

Spencer-Brown consistently holds fast to his idea of introducing symbols that, in multiple meanings,  can be both an operation and the result of this operation. The multiple meaning of *f* is carefully introduced and bound to clear rules. It contains within itself a contradiction, or—in the language of the early twentieth century—a category mistake: *f* is, as the course-of-values of arguments *a*, both a set of elements and is at the same time to be equated with the arguments *a* through re-entry and admitted among them. This corresponds to the situation of Russell's antinomy, which speaks of the set of sets that are negatively self-referentially defined (the set of all barbers who do not shave themselves, or mathematically: the set of all sets that do not contain themselves).

## – Re-entry and Re-insertion

This understanding of re-entry makes it possible to recognize the difference from re-insertion. Spencer-Brown uses the term re-insertion in the section on crosses and markers (LoF, 65), though in a different context. I use it here in a more precise sense.

If the technical course of a re-entry is described by means of the echelon, then a formally repeating operation occurs within it: in each iteration step the preceding figure is taken and appended at the end. This is re-insertion, whereby in the example of the infinite echelon the element  is continuously re-inserted.

By contrast, I understand re-entry as the operation that goes beyond the infinite sequence of re-insertions: for this sequence, which can only be indicated—as in ordinary mathematics—by the three dots ›...‹, it introduces a new expression *f*. *f* is not a further element of the sequence, but the *form of the sequence as a whole*. In formula E1, *f* stands for the infinite process of this re-insertion, but is not itself the element that is re-inserted. If *f* is admitted into the original set, that is the re-entry.

Note: André Oksas came upon a similar theme in 2024. He distinguishes between a “recursive” and a “sequential” interpretation of re-entry, and argues that for him only the sequential interpretation is

compatible with Spencer-Brown's modulator function. Here, by contrast, a third perspective is to be opened: whereas Oksas distinguishes two readings for the technical course within re-entry, the distinction proposed here concerns two different operations: iterative re-insertion on the one hand, and the symbolic-constitutive admission of a new expression into the calculus (re-entry) on the other. It is planned to return elsewhere to the different interpretations of the modulator.

## – The Idea of Proof according to Gödel

Spencer-Brown mentions Gödel already in the introduction (LoF, xix) and again in the notes (LoF, 96). Gödel's *idea of proof* (intuition behind the proof) for the incompleteness theorem can be regarded as a precursor to viewing a number at the same time as a function and a function as a number. Gödel showed how, in every sufficiently complex formal system in which both addition and multiplication, and over and above them prime numbers, are introduced, every sign can be interpreted both as a number and as a function, once the function is Gödelized. Gödelization is a translation: by means of a procedure based on prime numbers, a number is assigned to every letter and every punctuation mark, and thereby a unique natural number, the Gödel number, is assigned to every complete proposition composed of letters and punctuation marks. (Spencer-Brown convincingly emphasizes that this consideration does not apply to his calculus, since addition, multiplication, and prime numbers are not introduced there in a comparable way to the natural numbers; LoF, 96; AUM, Session Two.)

Gödel considered the self-contradictory proposition ›this proposition is false‹ and determined its Gödel number. Thereby it is admitted *as a number* into the formal system, although it contains an inner contradiction. His procedure can of course also be applied to every other proposition, such as the proposition ›n + 1 is the successor of n‹, and to all Peano axioms, with which the properties of the natural numbers are determined. By this method, statements about the totality of the natural numbers have become elements of the set of natural numbers.

Both with re-entry and with Gödelization, functions and arguments can be transformed into one another. Nevertheless, the differences between Gödel and Spencer-Brown are clear:

– In Gödelization, the transformation is brought in from outside. Gödel has found a translation with which every proposition can be translated into a number and, conversely, the number into the proposition. Spencer-Brown, by contrast, wishes to show how, from the process of the transformations forming an infinite series, the function *f* emerges as a limiting process.

– The Gödel number is unique and does not lie outside or between the known natural numbers, but is a natural number whose place on the number line is uniquely determined. For the symbol *f* newly

formed in Spencer-Brown, by contrast, there is no place, and its re-entry changes the medium in which the forms are entered.

How does Spencer-Brown solve the problem? If there is no number upon which the re-entry can fall and no place to which it can be uniquely assigned, then the re-entry can occur only by changing the properties of the medium in which all numbers are entered.

Spencer-Brown can formulate the result of re-entry only negatively: it is indeterminate in the given calculus where it can be admitted ("indeterminacy," LoF, 57). In order to find the fitting words, Spencer-Brown designates the transition from *a* to *f*, in greater generality, as the transition from an expression *e* (for *expression*) to an expression *e'* that is to be admitted through re-entry:

> "We have just introduced to *e'* a degree of indeterminacy in respect of its value which is not (as it was in the case of indeterminacy introduced merely by cause of using independent variables) necessarily resolved by fixing the value of each independent variable. But this does not preclude our equating such an expression with another, provided that the degree of indeterminacy shown by each expression is the same." (LoF, 57)

*f* wanders about, as it were, and finds no fixed place. This wandering leads Spencer-Brown to the oscillation with which he introduces the concept of time (LoF, 62-64). It is dissolved only when he passes from the wave-like course-forms to the networks of the modulator function: in these networks a way is found to modulate the arguments *a* into functions *f*. (See on this the paper for *The Unmarked State Conference celebrating the 50th Anniversary of the First Publication of George Spencer-Brown's 'Laws of Form'* in Liverpool, 9 August 2019).

In summary: Gödelization is an external coding: a unique natural number is assigned to every proposition. This remains a static assignment. In Gödel, the distinction between function and argument is preserved. The proposition becomes a number, but the number is not a proposition. The back-translation of the number into the proposition is guaranteed by the inverse function of Gödelization. And if the number is not merely to be read but executed as an instruction, a further instance is required – similar to hacking, where numbers become executable programs and for this require an interpreter (software that processes program instructions line by line). In Spencer-Brown, by contrast, the distinction between argument and function is transcended through re-entry: the function *f* arises from the arguments *a* without being reducible to them.

# The Sacred Middle of the Echelon (according to Kunze)

For Spencer-Brown, re-entry is the operation by which paradoxes and antinomies can be resolved. It is intended to lead from a logic that runs into paradoxical situations and unsolvable puzzles to a

logic in which negative self-reference is possible. Regardless of whether my interpretation is correct —namely, that Spencer-Brown intends in this way to resolve the transition from arguments *a* to the function *f* and thereby settle the open questions posed by Frege, Russell, and Wittgenstein— Spencer-Brown understands his own line of thought as a conclusion that leads, via re-entry, from a parasitic logic to a creative logic (LoF, 102).

If that is the case, one might ask what characteristics enable re-entry to fulfill this mediating function. It must involve more than a technical formalism that can be described as a ladder and requires only a parasitic logic. This is the question addressed by Donald Kunze (* 1947), who taught architectural theory and general art criticism at Pennsylvania State University and whose work focuses in particular on the intersection of architecture and Jacques Lacan's psychoanalysis. He seeks to show that, for such a conclusion to succeed, its mediating element must be elevated to the status of a saint.

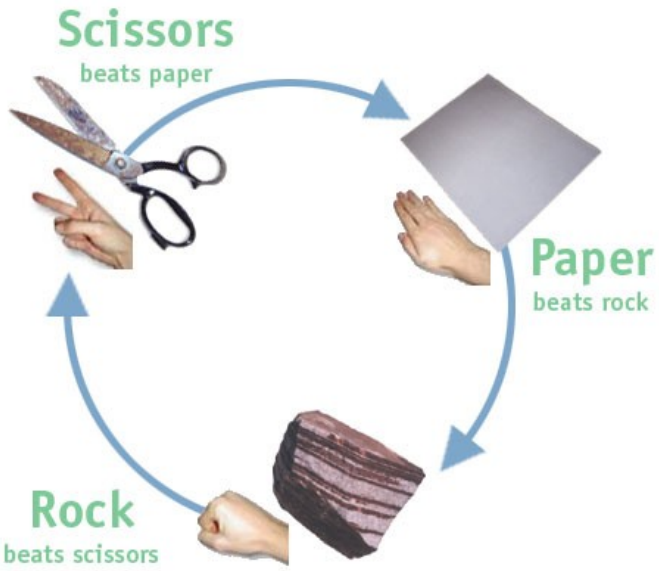
Spencer-Brown's commitment to Buddhism and his accounts of an enlightenment are not personal quirks, but the expression of an insight that is constitutive for his logical work. This has two far-reaching consequences that extend beyond traditional logic and are therefore often met with resistance in academic teaching: the insight gained at re-entry holds for all successful inferences and allows them to be understood anew; and the understanding of the sacred – both as the process of sanctification and as the result of the sanctified – is redefined in contrast to the tradition of the Enlightenment and positivism. The sacred no longer counts as something otherworldly that stands over against rational thought, withdraws from it, and calls it into question; rather, it shows itself at and with the operations of rational thought itself as the structure of mediation that enables the transition from the arguments to the function.

Outline: First, it will be reconstructed how Kunze argues technically: he understands re-entry as the oscillation of transitivity and intransitivity. Following this, his examples from the history of art, architecture, and the city will be discussed. Finally, the question will be asked what follows from these examples for a new understanding of the sacred, and in what way this understanding not only illustrates the logic of re-entry but grounds it. With Kunze, the sacred side of both the processes of proving and of counting will be determined.

## – What Is Intransitivity?

What is distinctive about intransitivity? In everyday life we usually compare things that are uniformly ordered and can be measured against a common standard (*etalon*), of which they are multiples. If, for example, the lengths of bolts of cloth are compared, they can be measured against the ell. They are transitive: if b is greater than a and c is greater than b, then c is all the more greater

than a. This no longer holds in the case of intransitivity. The simplest example is the game *rock, paper, scissors*: although the scissors are stronger than the paper that they cut, and the rock is stronger than the scissors that it blunts, the rock is nevertheless weaker than the paper by which it is wrapped up:




**The figures of rock, paper, scissors**

Source: wikimedia

When children play this game for the first time, they are astonished, at first do not understand what is actually happening here, and are then enthusiastic about it and can repeat it endlessly, as if they wanted to test whether it might not after all at some point turn into a transitive order. What they learn here is an example of mathematical creativity (cf. Spencer-Brown in AUM, Session 2). Intransitivity contains something that goes beyond ordinary mathematics: it is cyclic. An intransitive order returns to itself. And it is obviously non-hierarchical: every hierarchical ascent is a transitive order: after the stones come the plants, after them the animals, then the human being, and at the highest level, depending on one's belief, God, spirit, or the Absolute. In a cyclic order, the highest level returns to the lowest ground. (Every revelation bears intransitive traits, when, for example, God shows himself in the form of a lily or in a burning bush.)

This return is what Spencer-Brown has in mind when he seeks to incorporate re-entry into the medium (the foundation of forms) rather than into a higher form of forms. While we have thus far discussed the language of re-entry, it will become clear how intransitivity and cyclicity constitute the mathematics of re-entry. From this perspective, the significance and impact of the re-entry introduced by Spencer-Brown can be explained in relation to the concepts of heterarchical thinking, cybernetics, and even Luhmann's conception of a new sociology (theory of society), as well as a

new understanding of imaginary numbers and analogy—to which this text on re-entry in Spencer-Brown's work is intended to contribute.

## – The Two Sides of the Echelon

Up to now, in consequence C7 $\overline{\overline{a}\,b\,\overline{c}} = \overline{a\,c}\;\overline{b\,c}$, exclusively the left side has been considered, in order to develop from it the infinite echelon. But what about the right side and its two partial segments, with which *c* seems to double itself and to push between *a* and *b*?

Kunze follows Spencer-Brown's demonstration. In the first step, in the echelon shown on the left, the *b* standing in the middle is replaced by the recrossing $\overline{\overline{b}}$. Thereby it becomes possible, in the second step, for *c* to be doubled and distributed both next to *a* and within the recrossing $\overline{\overline{b}}$ into the space between the two boundaries. In this second step Kunze sees a cyclic, intransitive movement of *c*, so that consequence C7 as a whole equates transitivity and intransitivity and oscillates between them in the processes of breaking (*break*) and making (*make*). In what follows, this will first be transferred to a famous drawing by Dürer, in which he self-reflexively draws what perspectival drawing is, and then to the interpretation of sacred sites in Rome, which for him stand representatively for the emergence of metropolises. What at first looks like a clever idea of proof thereby acquires for him a significance that goes far beyond mere logic.

However, somewhat confusingly, he renames the variables *a*, *b*, and *c*, written in lower case by Spencer-Brown, as capitals A, B, and C. In mathematics, variables are usually represented by lower-case letters and constants by capitals.

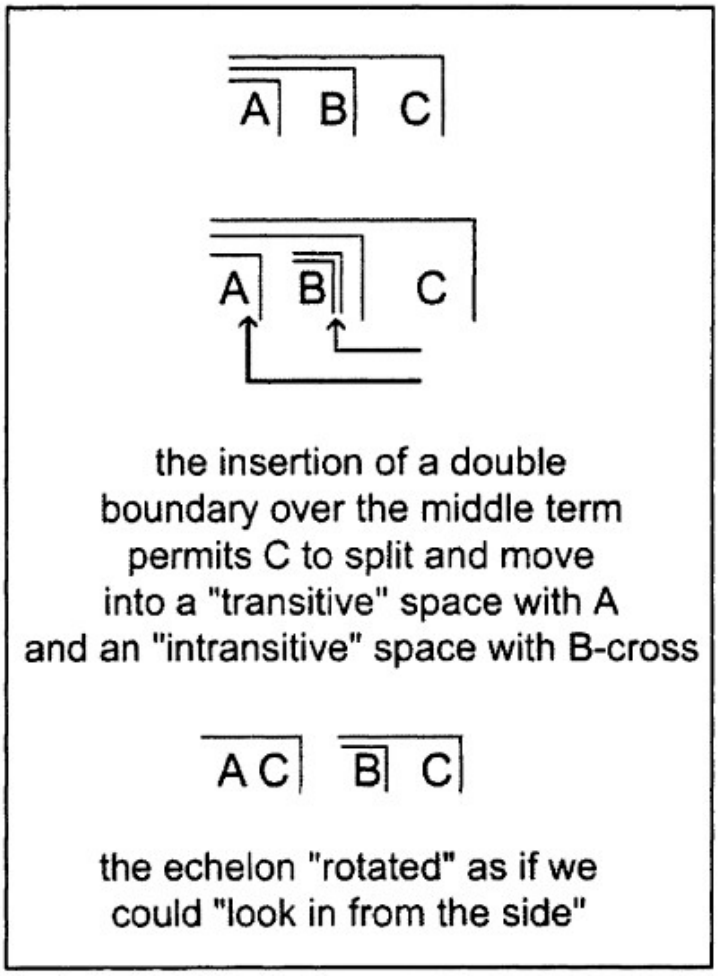



Fig. 2. The "rotation" of the echelon expression.

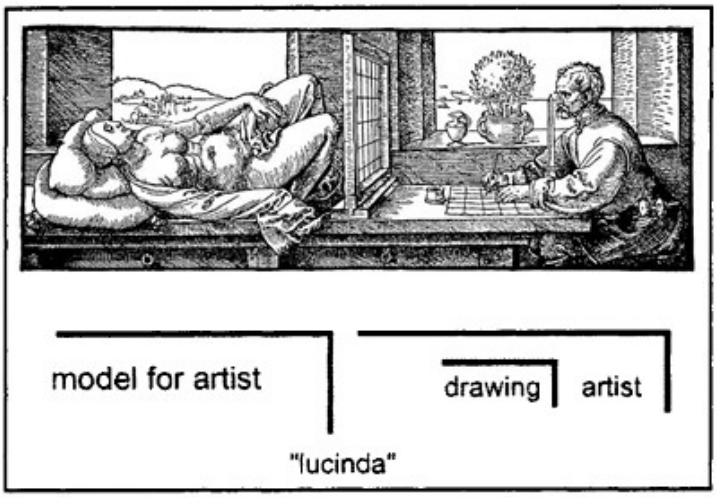



Fig. 3. Dürer's "Artist and Model in the Studio" as a "rotated expression."

We must begin with the interpretation of Spencer-Brown's demonstration. The demonstration of consequence C7 is substantially simpler than that of Canon C9, whereby the idea of proof emerges more clearly. I go back to the original in Spencer-Brown (LoF, 34), add there in red the cyclic movement emphasized by Kunze, and therefore switch provisionally to the lower-case letters chosen by Spencer-Brown:

**Consequence 7. Echelon**

C7 $\overline{\overline{a\,|\,b\,|\,c}} = \overline{ac}\,|\,\overline{b\,|\,c}$ break ⇌ make

*Demonstration*

$\overline{\overline{a\,|\,b\,|\,c}}$

$= \overline{a\,|\,\overline{b\,|\,c}}$ C1

$= \overline{\overline{ac}\,|\,\overline{b\,|\,c}}$ J2

$= \overline{ac}\,|\,\overline{b\,|\,c}$ C1.

C1: In the first step, consequence C1 is used, which Spencer-Brown designates as reflection (LoF, 28): if one passes from a variable *a* into its exterior and then again from the exterior into the exterior of the exterior, that is, in the result returns in a cyclic movement to the interior, this is a generalization of arithmetic and logical double negation: from a = - (-a) or a = ¬ (¬a) (*a is not not-a*) there results: *a* = $\overline{\overline{a}}$. If this consequence C1 is applied to the *b* standing in the middle, then it can be replaced by $\overline{\overline{b}}$.

J2: In the second step, the *c* on the far right is distributed according to transposition rule J2 over the two characters to the left of *c*, i.e., over both *a* and $\overline{\overline{b}}$. Here it is decisive that it does not come to stand *outside* the double boundary of the going-forth and returning of the recrossing $\overline{\overline{b}}$, but *between* the two boundaries surrounding *b*. It stands *in the process of the double negation of b* (recrossing of *b*).

C1: Third step: now the sign on the right side is surrounded by a double boundary, which in turn can be omitted according to C1. Thereby the desired result is reached.

For Kunze, with the replacement of b by $\overline{\overline{b}}$, the sanctification of *b* has already technically occurred, because with the two boundaries of the “crossing once more” (of the “re-crossing,” *recross*, LoF, 2) there arises between the two boundaries the space that will become the sacred place. “Spencer-Brown's double bound creates a 'pocket' space (A B[] C)” (Kunze, 333). In order to see here more than a mere application of consequence C1, the analogies with Dürer's drawing and with the founding myths of Rome, developed in what follows, are required.

It is expressly necessary to point out a peculiarity: in a pure cyclicity, a-b-c would, after the three steps (rotations) a-b-c, b-c-a, and c-a-b, return in the fourth step to a-b-c itself. In consequence C7, however, in the return from *c* to *a*, there arises the second term $\overline{\boldsymbol{b}}$c, which acts like the back (or night) side of the return from *c* to *a*. The relation of these two terms is the open question with which what follows is concerned. They will prove to be twins, one of whom stands in real life and the other of whom has descended into the underworld or realm of the dead.

## – A Change of Perspective in the Example of Dürer

The Draughtsman of the Reclining Woman (*Der Zeichner des liegenden Weibes*) by Albrecht Dürer (1471–1528) was included in 1538, after his death, in the second edition of his *Underweysung der Messung mit dem Zirckel und richtscheyt*, first published in 1525. Dürer proceeds quite deliberately, step by step and pedagogically: he begins with highly complex, almost purely mechanical experimental arrangements and ends with the most important tool for the artistic practice of perspectival drawing: the drawing frame (also called velum, Italian *velo*, literally translated 'sail, cover, veil'). Its precursor was introduced and presented in 1435/36 by Leon Battista Alberti (1404–1472) in his treatise *De pictura* (On Painting). For Alberti, the velo was a relatively simple, loose cloth interwoven with a grid. It is hung in front of the object to be drawn. If the painter paints section by section what he sees in the individual squares of the cloth or grid, the perspectival laws are satisfied. Dürer, by contrast, thought like an engineer: he made solid, precise wooden constructions out of it, with fixed marking needles, weights, and hinges, which craftsmen could precisely reproduce.

The velo described by Alberti is an intermediate layer (like a semi-transparent membrane) that served as an interface between the three-dimensional world and the two-dimensional canvas. This admittedly restricted the free view of the object to be drawn, but simplified the perspectival drawing. The cloth or grid remained transparent enough to be able to see the object – albeit less sharply. With it, it becomes easier to reproduce a three-dimensional figure or landscape on a two-dimensional sheet. Dürer extended this technically, and later possibilities such as photography and its grain continue this.

The idea suggested by Kunze is now that Dürer does not merely draw perspectival drawing, but that with this drawing a transition occurs from a classical sign (the imitative representation of an object) to a technical sign (the *flow diagrams* of a system). With Dürer's technical drawings, what is no longer simply depicted is a static object, but the *process* of drawing in all the steps of perception, apprehension, and reproduction. The viewer understands what is happening here and can apply it in his own painting practice.

This is a *change of perspective of the second kind*: what is not gained is a new perspective by walking around a fixed object and seeing it now from the front, now from above, or from the side; rather, the focus shifts from the fixed object to the dynamic, technicized process. Dürer no longer merely draws the woman, but he draws the *process of drawing* itself. It visualizes an operational system (eye/sight → grid → coordinate transfer → paper). The drawing becomes an algorithmic instruction manual, a flow diagram of the emergence of the picture.

Kunze recognizes in Dürer's drawing the right side of consequence C7: here A stands for the woman to be drawn, C for the artist, and B for the drawing *process* (*drawing*). In the drawing process (B), with the drawing tool (the drawing frame), a distance initially arises between the model (A) and the artist (C), by means of which, however, the perspectival drawing succeeds.

Kunze leaves open why both AC and $\overline{B}$C are in turn negated once more. Strictly speaking, AC is the *appearance* as the artist C sees the woman A lying before him. The negation of this appearance is – in Kantian terms – the thing-in-itself of this appearance, that is, the woman herself, whose appearance the painter sees. – And in $\overline{B}$C, C in turn stands for the artist, but $\overline{B}$ is the negation of the drawing *process* (of the *drawing*, B). I understand this to mean that with this negation of the drawing process, what is meant is the respectively static state of the drawing achieved at a particular point in time, concretely in the moment in which Dürer has captured the drawing *process* in his drawing of the drawing. In order to be able to draw it, he must *freeze* this moment of the process. When Dürer's drawing is viewed, it looks as though the draughtsman is just beginning to draw and has perhaps set the first point. If $\overline{B}$C is now in turn negated, then it is the *painting* itself that is completed as a work beyond the respective drawing step and the artist. This painting in turn preserves the truth of the model A as it was seen by the artist C and shaped in the drawing B.

After these “preliminary works,” the question decisive for Kunze can be posed: can it be understood with and upon Dürer's drawing what happens in the transition from B to $\overline{\overline{B}}$, and why this can be seen as a sanctification of B? Now it is no longer merely a matter of the change of perspective of the second kind to the drawing of a process, but of how this process can succeed, that is, successfully convert an A into a C, only if it rests inwardly on a continuous oscillation of concealing and revealing. The artist C, while drawing (B), looks up alternately from his painting and at this moment interrupts the work in order to look at the model A and take up its appearance, and then again interrupts the eye contact with A, looks at his emerging painting, and continues to work on it. If the entire drawing process B is understood as the mediation of how an inference can be made from the model A to the artist C, who produces the painting of A and with it hits the character of A, then the mediating step B, as the oscillation of looking-toward and looking-away (looking at one's

own emerging drawing instead of at A), becomes dynamic in turn. With the eye contact, the artist opens himself to the model; by contrast, the concentration on drawing is a withdrawal into itself (*Insichgehen*), with which the artist turns away from the model. In summary: just as a change of perspective from a drawing of static objects to a drawing of dynamic processes had succeeded, so within this change of perspective there occurs a change from a static mediating link B to a dynamic  oscillation of turning-toward and turning-away.

With the insight into this novel change of perspective, the decisive step has succeeded, in order finally to understand, with the references to the history of the emergence of metropolises, why this change of perspective can be interpreted as sanctification. In the religious-studies and philosophical sense (for instance in Giorgio Agamben or in the ancient concept of the *sacrum*), “to sanctify” or “to sanction” means drawing an inviolable boundary that separates a space from profane everyday life in order to make a higher truth visible within it. This is the theme with which Kunze is concerned in his concluding step.

Thus it also becomes clear why Kunze uses this drawing by Dürer for his *Theory of the Metropolis*: the historical city (the metropolis of the nineteenth and twentieth centuries) functions exactly like Dürer's recrossing:

– It is no longer defined by its static city wall (the classical boundary), but by the permanent oscillation at its boundaries (commuting in, commuting out, flows of goods, the alternation of private and public spaces).

– The metropolis “sanctifies” urban space by completely mathematizing and dynamizing it through traffic networks, grid systems (like the drawing frame), and logistical processes. It makes movement the actual purpose of the city's existence.

Note. It takes the eye of an architect to understand what is happening here. Usually drawing is represented from the perspective of the draughtsman: in the foreground stands or sits the draughtsman, the viewer of the painting looks over his shoulder and sees from his perspective both the object to be drawn and the emerging painting, so almost paradigmatically in Jan Vermeer (1632 –1675): he paints self-referentially not simply a picture, but a picture with the title *The Art of Painting*.



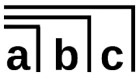
**Vermeer:** ***The Art of Painting*** (1666/68)

Author: By Jan Vermeer - lAHeqBoLaePtEA at Google Cultural Institute maximum zoom level, public domain, Link

Whereas in Dürer the woman to be drawn lies horizontally on the picture plane, in Vermeer she stands upright facing the painter. Dürer has a naked woman drawn who conceals her shame in the gesture of the Venus pudica, whereas in Vermeer both persons appear in bourgeois, everyday clothing and the woman to be drawn, with lowered head and almost closed eyes, looks down somewhat shyly or devoutly. That Dürer's drawing was subsequently assigned the title *Emotivität des Weibes* [Emotivity of the Woman] admittedly rests on a mere algorithm error of automatic cataloguing when this drawing was recorded in zeno.org and was taken over from there into Wikimedia Commons in 2012, but it draws attention to an important detail. The expression 'Emotivity' arose around 1900 in psychology, psychiatry, and sexology and in technical language means an increased (often sexually intended) excitability of the mind. Today this expression is projected onto this drawing by feminist approaches in the context of the male gaze. This current debate of the twenty-first century, however, does not do justice to Dürer's intention: he was not

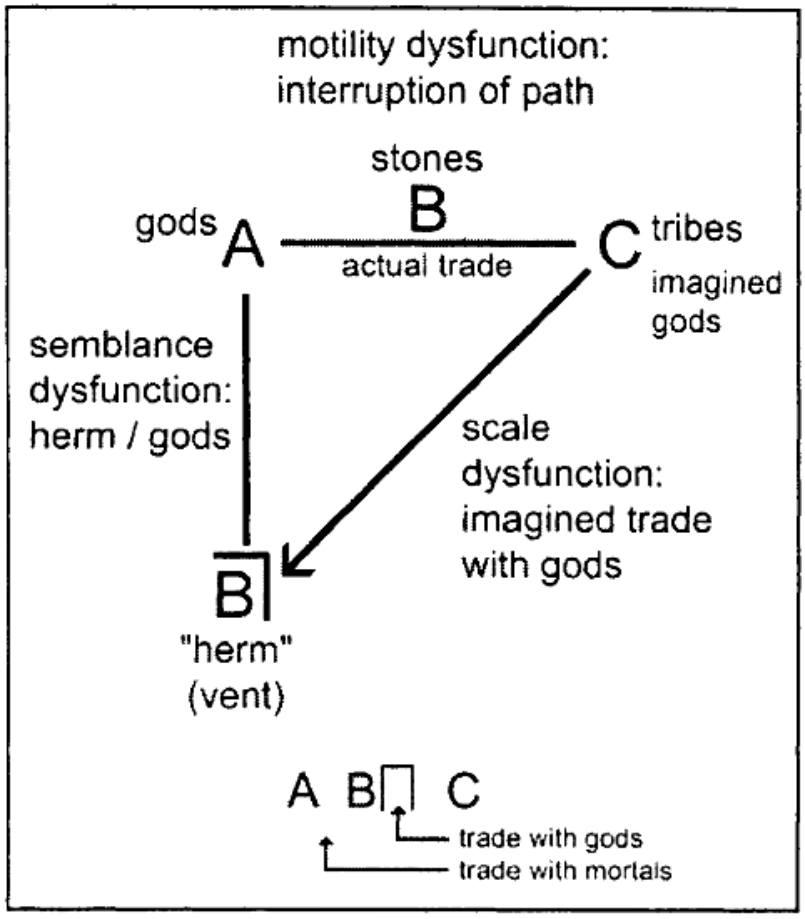
concerned with the disparagement of the feminine, but with the oscillation of concealment and unconcealment. He stages the relation of nature and art both immediately through nakedness and metaphorically through the medial barrier of the grid, which mathematically opens up the space in order to support perspectival drawing.

## – The Profane and the Sacred Level of Silent Trade

Cities are said to have historically emerged at crossroads that developed into trading centers. But it was not that simple. Originally, meetings and trade with strangers were frowned upon. Dwellings were, as in the form 

, separated from one another by boundaries, arranged transitively, one beside the other, without contact with one another. Each household had its own gods. Strangers were felt to be a danger. “Crossroads were still a danger in the time of Classical Greece; Oedipus meets and murders his true father at a crossroads.” (Kunze, 334) But people found a solution: some laid down their surplus at certain crossroads, which others later picked up and laid down their own surplus, so that they never had to meet in person. To mark these trading places, stones and small towers of stones were laid down or erected at them. These were the original herms, for Hermes, the god of trade, theft, and communication. These places were regarded as sacred sites, and around them the first metropolises developed in the course of time. (See the discussion in Wikipedia. Even if critics doubt whether this is historically accurate or based solely on myths and legends, it does not alter Kunze's argument. See also Baecker 2006, 100f).

Trade took place on two levels: on the one hand, it was a *profane trade* in physical goods that were laid down and picked up. On the other hand, it was something sacred, a *sacral trade*: whoever laid down his goods there felt this as a sacrifice to the god to whom this place was dedicated (sanctified). And whoever picked up goods there saw this as a gift that he had received from the god under whose protection the place stands. Kunze depicts this in a triangular figure that takes up and continues the cyclic movement of *c* in the demonstration of consequence C7. On the upper horizontal runs the profane trade, across the sacred place at the lower vertex the sacral trade. These two levels are separated by an inviolable boundary. Whoever violates this boundary is sacrificed or sacrifices himself in order to preserve and secure this order in its entirety.




**The “triangle” of silent trade** (“The 'triangle' of silent trade”), Kunze, 335 Fig. 5

Here, at $\overline{B|}$, the decisive place *between* the two boundaries surrounding B is highlighted and designated as “pocket” (Kunze, 333). At this place the sacral exchange occurs, which takes place via the protective gods of the trading places. From this perspective, the two partial segments of the right side of consequence C7 acquire a clear meaning: AC stands for the physical trade, $\overline{B|}$C for the trade with the protective god. Kunze emphasizes three aspects of this (Kunze, 334f.):

– The motility of direct trade and action is interrupted, in that both tribes avoid one another, but according to an indirect, possibly unspoken agreement lay down and pick up their goods at the same place, one after the other in time.

– The semblance of a direct trade is broken: according to their respective self-understanding, it is not the real tribes A and C who trade with one another at a real place B; rather, with the sanctification of place B, the products stored and exchanged there are also sanctified. Seen thus, the tribes do not exchange products *with one another*, but with the herm $\overline{B|}$.

– The scalar is broken: instead of a real trade in real goods, a sacred transformation occurs at place B: what one tribe has laid down receives a special meaning through the sanctification, which is why

the other tribe may pick up these sanctified products in the belief that it has not traded its product with the other tribe, but has received the products sanctified by the herm from the herm.

That this is not merely a subsequent, theoretical consideration is confirmed for Kunze in the foundation and spatial structure of Rome (Kunze, 335f.). The city developed around the seven hills; in the valley between them lay the place for trade with the respective outsiders. It formed neutral terrain between the settlements of the various tribes (e.g., Latins and Sabines) and was clearly separated from them. This place possessed a sacral character.

Romulus drew a further boundary and demarcated the city area (pomerium) by a sacred plough furrow (*sulcus primigenius*) and thereby founded his power. His twin brother Remus mocked this and jumped over the trench. Because of this breach of taboo he was killed and entered the realm of the dead. This myth explains why the neutral place of trade is at the same time connected with the cult of the dead and with cemeteries. The guardians of the trading place not only protected the trading place in the name of the protective gods, but also watched over the peace of the dead there. Romulus stands for order, law, legislation, light/the upper world, and the visible political space of the city. Remus stands for sacrifice, boundary transgression, the underworld/realm of the dead, and the foundation of the sacred. For Kunze it is important: the city does not rest solely on the legal order of Romulus, but is necessarily bound to the repressed/dead (Remus). The market/trading place functions as the interface at which these two spheres (upper world and underworld, order and chaos) meet and are mediated.

Kunze sees this confirmed by a further myth (Kunze, 335f.): when the ground of the trading place broke open through an earthquake or another force and an impassable abyss arose, the place could no longer be used. The citizen Marcus Curtius sacrificed himself for the welfare of Rome: he rode armed into the cleft down into the underworld, whereupon the earth closed again and the place became usable. “The artefact deriving from Curtius' sacrifice is visible today as a series of concentrically arranged stones known as the 'Curtian Lake' (Lacus Curtius). A small memorial plaque shows an armed rider descending into the underworld.” (Kunze, 335).





**Lacus Curtius on the Forum Romanum in Rome**

Source: (a) Wikimedia, ID 7223523, accessed 11 October 2026

(b) By Lalupa - Own work, CC BY-SA 3.0, Wikimedia, ID 37054225, accessed 13 October 2026

## – The Two Levels of Inference – Technical-Formal Course and Creative Idea of Proof

This opens a new understanding of logical inference from *a* through *b* to *c*, in which two levels are likewise to be distinguished. In its technical course, it is to be examined whether it is free of contradiction and remains comprehensible and perspicuous. With the sacral level, the concept is reached for which Wittgenstein struggled all his life but which he did not reach, since he consistently wanted to explain everything from the profane technical level and to remain silent about the sacral level: the idea of proof [*Beweisidee*].

If the echelon of *a*, *b*, and *c* is understood as a logical inference that leads from the premise *a* through the middle term *b* to the conclusion *c*, which represents something of a higher order compared to *a*, then this one-sided view can no longer be maintained. What no longer suffices is simply a transitive (linear, causal) inference that leads continuously to higher insights; rather, in accordance with the interpretation of the echelon in Spencer-Brown, the inference succeeds only if the middle term *b* is in turn understood as dynamic and in its oscillating permeability. Just as the drawing frame is grounded in and uses the laws of geometry, so a middle term is grounded in a calculus with which the premise *a* is decomposed by the grid of the calculus into its various components and aspects and reassembled for the inference to *c*. Only through this grid does the inference to *c* succeed *within* the calculus. The conclusion is bound to the calculus through *b*.

And just as the artist is able to shape a new view of the model *a* with his painting, so a new insight into *a* can succeed with the inference. Just as the artist can draw the model *a* only as it *appears to him in the drawing frame*, so the middle term *b* embedded in a calculus can see in the premise *a* only what it presents to the calculus. And just as the artistic drawing *process b* falls apart into the individual steps of drawing, in which the painter looks now at the model and now at his emerging

drawing, so the inference is composed of the individual steps of a chain of proof, in which one looks alternately at the task and at the emerging course of proof and its inner freedom from contradiction and perspicuity.

But only Kunze's distinction between profane and sacral trade gives the decisive hint:

– On the first level, a mapping occurs from the model *a* onto the painting *c*, or from the task *a* onto its solution *c*.

– On the second level, the inference proceeds and succeeds from the attractiveness of the model *a* to the creativity of the artist *c*, and shows how one can infer from the effect of *a* to the *artistic activity* of *c*, which is inspired by *a*. Correspondingly, here the inference succeeds from the open question given with the premise *a* to the *creative idea of proof* in the one who is able to carry out the inference.

This hidden inference to the idea of proof, which stands behind the technical execution of inferring and secures its success, addresses the *power of imagination* thanks to which the idea of proof becomes possible. Some call this intuition (inspiration) or intellectual intuition. It is compared to the lightning that leaps over and ignites in the moment when *a light dawns* on the one inferring. – If this does not succeed, a fallacy threatens that may look formally correct at first glance but misses the state of affairs. Jan van der Meulen spoke of the *broken middle*. In order to succeed, the middle must be carried by the second level.

This illuminates the great question with which Wittgenstein was concerned all his life: how can a linguistic expression accurately reproduce a state of affairs although it does not resemble it in its sensuously perceptible form? The word 'tree' does not look like a tree. A proposition like “The tree is green” does not look like the state of affairs that it describes. Already in the *Secret War Diaries* 1914–16, the question of a picture theory is adumbrated that goes beyond the traditional theory of depiction and gives propositions an autonomous autonomy thanks to which this is possible. “The proposition is a formation with the logical features of what is depicted and with yet other features; these latter will now be arbitrary and different in different sign-languages.” (22 October 1914, Ms-101, my translation).

What Wittgenstein is concerned with and for which he found no conclusive answer becomes clearer when, with Kunze, two levels are distinguished. On the first level, it is a matter of the inference from the given state of affairs *a* to the linguistic expression *c* (a word or a proposition) with which *a* can be accurately formulated. On the second level, it is a matter of the question how a state of affairs can at all stimulate the capacity for linguistic articulation: how from the encounter with an

event there emerges the possibility of expressing linguistically the state of affairs experienced therein.

Wittgenstein took up questions of this kind again and again, up to the manuscripts written in 1937–41, which were for the most part published posthumously under the title *Remarks on the Foundations of Mathematics*. But a conclusive answer was not forthcoming. He investigated why and when a proof counts as successful, and in ever new approaches speaks of the picture, the figure, the structure, the process, the construction, the technique, the perspicuity, and the persuasive power of the proof, without expressly making the question of the idea of proof into a theme. In the idea of proof it shows itself how the tension given with the open question discharges into a novel approach to a solution, which can subsequently be worked out technically in detail. It seems as though Wittgenstein were approaching the significance of the idea of proof for the proof as one approaches a limit, without reaching it.

Here Kunze's interpretation opens a possible access. It makes intelligible in what way the riddle can be grasped with the help of the “shadow side” of the echelon, which stands on the right side in consequence C7. When Kunze understands the second level as the sacral level, this seems to me to be that domain about which, for Wittgenstein, philosophy can only be silent (TLP 7).

## – Sacred Numbers of the First and Second Order according to Plato

Counting is at first glance a far less demanding achievement than proving complex tasks, and with the repetition of the always identical counting step it is almost mechanical. Precisely this, however, makes possible an insight that goes beyond the previous expositions and, in retrospect, allows the artistic and proving process to be better understood as well: in the counting process, the distinction of the two levels becomes evident as a *curvature* recognizable *within* the task, and with the *measure* of curvature a third value emerges with which Plato, and later not least Gotthard Günther and Spencer-Brown, were concerned.

Counting appears as an almost self-evident sequence of the natural numbers 1, 2, 3, …. But what happens in the transition from a number to its successor? The gap from one number to the next appears unimportant, something that can be overlooked. Here, by contrast, it is to be shown how a dynamic develops in and out of the gap, leading to a new understanding of numbers. In particular, two orders of sacred numbers will be distinguished that proceed from Plato and lead beyond him. This process from the first to the second order opens a new view of what happens in re-entry.

Counting does not look like an inference that leads from a number to its successor and even requires a special idea of proof. Objects are simply counted by assigning to them successive ascending numbers: the first, second, third, and so on. But what happens in the *course* of a counting step, in

the simplest case from one to two, and generally from *n* to its successor *n + 1*? What is the distance between numbers? It is skipped over by the inconspicuous operation *plus one*. Is it possible to describe this operation with the figure presented by Kunze? Then *b* is the crossing of the gap between the two adjacent numbers *a* and *c*. The operation *plus one* corresponds, as a counting process, to the drawing process or to the process of proving inference. Here a distinction must be made: the number *n + 1* is the successor of *n*. The difference of the two numbers ((n+1)-n) amounts, in contemporary arithmetic understanding, always to 1. Geometrically thought, this is the shortest connection of a number to its successor. The operation through which the successor is brought forth is neither the number *n* nor the number *n + 1*, but a rule of transition.

What properties does the distance have? They are to be described step by step as straightness (*artiotes*, the linear framework), intervality (*diastematotes*, the measurable extension), and indeterminacy (*aoristía*, metaphysical openness, potential field), from which the capacity for curvature (*kýmansis*) emerges. The capacity for curvature generates the tension from which re-entry can emerge, and opens the inner degrees of freedom thanks to which re-entry can return.

– **Straightness (*artiotes*)**. Plato interweaves the two aspects of numbers and their distances. He expressed himself in various contexts not entirely consistently, and important components of his doctrine were transmitted only orally and count as unwritten doctrine. Hence there are various interpretations. On the whole, I follow the expositions of Stenzel. For him, two principles exist in Platonic arithmetic: the countable units and the distances (intervals) between them, which are likewise understood structurally as numbers. The actual, form-giving units are, for him, the odd numbers. These act like isolated points that are entered into an unlimited medium, while the even numbers stand for the intervening distances.

It is surprising, however, which names Plato gives to these numbers. The odd numbers are *perissós*, literally: the 'excessive, superfluous, useless, extravagant' (Wilhelm Pape, cited in LSJ, refers to Plato Rep. III, 407 b, Prot. 356 e, and Polit. 282 c). Here, however, 'superfluous' or 'useless' is by no means meant pejoratively, but on the contrary: when a number is divided into two halves, something always remains over in the case of odd numbers. For example, the odd number 9 can be divided into 4 + 4 + 1. The two fours are the largest possible halves, and the 1 remains over.

Hence, for Plato, the odd numbers are those that first confer on a thing its characteristic form and boundary. Here the remaining portion stands symbolically for that which – thought further – survives all material changes and, like the soul after the death of the body, remains pure and unchanging (Phaedo 103c–106e). Contrary to the first linguistic impression of 'superfluous,' the odd numbers in Plato thus prove to be the actual, sacred numbers.

It is exactly the reverse with the even numbers. Plato chooses the designation *artios*, which are literally the 'fitting, suitable, perfect' numbers. Frequently cited passages: Gorgias 451a–c, Theaetetus 185d, Philebus 24a–25b. They form the *in-between* with which the distinction of two numbers and their distance from one another first become possible. *artios* sounds at first glance far more positive than *perissós*. Even numbers can be divided without remainder into two halves, but precisely herein lies their metaphysical weakness: they lack the middle that distinguishes itself over against the two halves.

In this sense they are the filling material from which the odd numbers can first be designed. They form the linear framework: uniform and rigid within, while the spirit first awakens with the characteristic form of the odd numbers. This in-between functions as the supporting medium that first holds apart the firm, punctual structure of the odd monads; straightness spans the first basal coordinate system. Nevertheless, for Plato they are the principially transient, from which the non-transient of the odd numbers differs substantially. In what follows, however, it will become apparent how precisely from this seemingly inferior filling material and the mere gaps that are mostly overlooked in classical counting, an inner dynamic arises. It is this dynamic that will make the sacred numbers of the second order possible.

– **Intervality (*diastematotes*)**. The first even number is 2 with its two halves 1 + 1. It is not only the successor of the number 1 on the number line, but with it the distance is at the same time defined with which two adjacent numbers are separated from one another. In the *Timaeus*, Plato describes the composition and proportioning of the world soul with the help of mathematical ratios and harmonic intervals. He understands there the intermediate spaces between the numbers explicitly as *diastema* (*Timaeus* 36a) in the multiple meaning of 'intermediate space, distance, interval.' This can be generalized to the distances between numbers in general.

The distance has in turn two aspects: it is a spatial distance between two numbers that determines the in-between in which no further numbers lie within the natural numbers; and it has a value – the length of the distance, which in turn amounts to 1. With every new number this distance is passed on, when every number *n* is followed by a new number ›*n + 1*‹ that is in turn separated from *n* by a distance of length 1. It holds to the measure that is given with the first distance between the numbers 1 and 2.

Consistently, for Stenzel in Plato the first odd number is only 3, which follows 2, while 1 has no distance over against itself. Unlike in modern mathematics, according to Stenzel, for Plato the 1 is not odd, but stands as a superordinate origin outside the distinction of even and odd. The 1 is absolutely indivisible, while the even numbers can be divided into equal halves and the odd

numbers only into unequal portions (Stenzel, 58, 103 n. 3 with reference to Parmenides 143c–144a; see also Phaedo 104a–b and Aristotle, *Posterior Analytics*, Book II, ch. 13, 96a–b).

If one recalls Kunze's expositions on the founding myths of Rome, then intervality stands archaically for abysmalness – the opened trench (*mundus*) that leads directly into the realm of the dead. This already gives an intimation of how, with intervality, the shadow sides are struck that lie hidden on the dark reverse side of simple, linear counting.

– **Indeterminacy (*aoristía*.** The previously defined distance *b* transforms itself here in the sense of Spencer-Brown into a field of recrossing. From *b* there arises $\overline{\overline{b}|}$. In Plato this dimension becomes recognizable in that the even numbers at the same time stand for the unlimited (*apeiron*) and its immanent abysmalness. This is admittedly not directly transmitted in writing by Plato himself, but is anchored in his unwritten doctrine, which Aristotle reproduces very clearly in the context of his critique of metaphysics (Met. 987b; cf. on this Deutscher, 18).

For Aristotle, the even numbers thus refer to the principle of matter (*hyle*), which is universal (*katholou*) and essentially indeterminate (*aoristos*, Met. 1087a17), while the odd numbers represent determining form. George Spencer-Brown has radically formalized this intuition. Read thus, his systems-theoretical distinction into medium and form can be understood as a precise evolutionary reinterpretation of the Platonic difference of even and odd. With the concept of matter, a new quality is added: matter is not merely the passive medium from which the forms emerge and into which they are entered – as it were, in-formed – but rather forms the plastic substrate from which the forms are first made. A forthcoming investigation is to deal expressly with this.

In this indeterminacy interpreted by Aristotle as matter, the creative potential lies hidden from which everything new can arise. There now remains only a single step to take: to show how this indeterminate potential is able, thanks to its own capacity for curvature, to billow up in order thereby to open a completely new dimension – a dimension that leaves and transcends the hitherto developed two-valuedness of even and odd numbers, shakes it in its foundations, and yet is able to return into it again.

– **Capacity for Curvature (*kýmansis*)**. kýmansis designates the genuine process of wave-beating, the surging, the dynamic billowing-up. What is expressly not at issue here is a passive *kampylótes* – that is, a purely static crookedness that is imposed on a rigid series from outside (like the mechanical bending of a wire) – but the immanent capacity for curvature in the sense of a fluid capacity for self-movement and wave-formation. The curvable possesses an inner elasticity and an immanent dynamic (tension).

This tension breaks open the linear one-dimensionality of the number line: the distance “surges” or curves out of the line into a higher dimension. Only through this plastic curvature does the topological space (the bending) first arise at all, from which a movement can emerge that leads operatively to the movement of re-entry and its return (re-entry in the narrower sense).

Although this is not an established technical term of classical ontology, Aristotle provides the fundamental image in *On the Motion of Animals* (*Peri poreias zoon*, *De incessu animalium*) when he describes how snake-like beings move forward by their own power. He expounds there that they move forward by beating waves, and recognizes herein a dynamic, immanent form of curvature (*kámpsis*) that comes from the body itself in order first to generate movement in space: “but movement in wavy lines consists in self-curving” (*he de kýmansis kámpsis estín*, 709a28–29). When modern Greek physicists write about the wave-particle duality of Huygens and Einstein or about quantum mechanics, they consistently translate “wave movement” or “electromagnetic wave” as *elektromagnetike kýmansis*.

As fluid surging and wave-like self-curvature, *kýmansis* offers precisely that mathematical elasticity within the *diastemata* that allows space to return into itself. The *kýmansis* of the medium thus proves to be the fundamental ontological condition for the re-entry considered by Spencer-Brown.

– **Two Orders of Sacrality**. In Plato, the odd (*perissós*) represents the sacred, the static, the pure boundary – the principle resting in itself, transcendent. It persists in its sublime indivisibility. By comparison, the even (*ártios*) counts for him as the profane, the material, the infinitely divisible, the feminine. Because the even, however, is at the same time the constituting principle of movement, space, and extension, paradoxically only on this side lies the genuine capacity for fundamental transformation.

The odd cannot develop out of itself, since it is ontologically already finished, completed, and bounded. The dynamic evolution from *artiotes* (straightness) through *diastematotes* (intervality) and *aoristia* (indeterminacy) to *kýmansis* (capacity for curvature) thus takes place exclusively on this seemingly lower, material side. The continuum of distance must be fluid, plastic, and feminine-indeterminate in order to be able at all to billow up and curve. Thought in analogy to Hegel's master-slave dialectic: the sacred of the second order is here not effected by the static, pure “master” (the completed odd), but breaks forth as a dialectical reversal out of the inner, suppressed dynamic of the “slave” – that is, of matter, space, and distance.

In Plato, the decisive impulse in this direction is already laid down where he investigates the operative transcendence of the distance from one number to the next. If one thinks this thought radically further, a Copernican turn takes place: the distance is no longer the ontologically

insignificant that is to be skipped over thoughtlessly in counting. Rather, with distance the multiplicity and inner freedom are first given that transcend linear counting and lead it over into an operative re-entry. Re-entry is a movement that explodes the classical one-dimensionality of counting.

For this creative curvature there exists a quite proper, immanent measure. It manifests itself in the specific course-form of those curves that detach themselves from the linear stretch and are able to return to it again. Just as with the insight into the status and scope of the idea of proof the question could be clarified that remained open in Wittgenstein, so this measure of curvature shows that fundamental third value that Plato and later thinkers such as Gotthard Günther and Spencer-Brown sought. This third value is not a single, isolated numerical value; as a structural value it forms a completely new axis on which the measures of curvature can first be determined.

Mathematically this leads into the theory of fiber bundles, in which connection, path (trajectories), its curvature, and torsion are differentiated, and with it into quantum field theory. In the result, this reversal generates in quite different domains of mathematics those novel entities that exploded the familiar system of the natural numbers and brought the mathematics going back to Pythagoras into a crisis: the square root of two ($\sqrt{2}$ as the diagonal in the unit square), the golden ratio Φ (as the proportioning of two diagonals in the pentagram ☆), the circle number $\pi$ and the Eulerian constant *e* linked with it through the Euler formula ›$e^{i \cdot \pi} = -1$‹, as well as the imaginary unit *i*.

The wave valleys and gaps billowed up by *kýmansis* thus create the topological space in which these transcendent numbers can settle as sacred numbers of the second order. With them the mathematical number line reaches those new orders of magnitude (the continuum) that Georg Cantor distinguished from the countability of the natural numbers.

## Galois Theory as the Mathematics of Re-entry and of the Proemial Relation

Spencer-Brown has introduced, with the mark ⌐, an elementary sign that *in itself* bears the tension-filled and creative double mode of element and operation. But there is in his work no mathematical formalism that proceeds from the *inner dynamics* of this dual nature and leads to re-entry. Instead, re-entry is somewhat improvised from a successor relation given with the echelon and loses the “connection with arithmetic” (LoF, 58; on this Kaehr, 42). Gotthard Günther, coming from a different direction, hits precisely this point when, with the proemial relation (literally: a preceding justificator), he considers philosophically how argument and relation dialectically exchange their

places and how new orders can thereby emerge. But he too leaves the mathematical realization open.

This succeeds with Galois theory, which goes back to the French mathematician Évariste Galois (1811-1832). It describes the extension of number sets algebraically as the relation of ground field to extension field, and thereby providing a model for the shift between levels of order. At the same time, Galois recognized the intrinsic symmetries inherent in this development. These symmetries make it possible to unfold the self-referential nature of function and argument without falling into Russell's antinomy. The example of the introduction of imaginary numbers will prove to be the mathematical paradigm.

In what follows, the proemial relation will first be introduced and motivated, before it becomes technical with Galois theory. But the result is worth it: it shows how, with Galois theory, one of the most significant transformations of modern mathematics has succeeded: the transition from a mathematics of numbers to a mathematics of *paths of solution*, which can be made fruitful for the understanding of both Spencer-Brown and Günther, and lets it be recognized how they mutually illuminate one another. (I owe the decisive technical hint to a chat with Deepseek in October 2026, which, in response to my questions about Spencer-Brown and the proemial relation, recognized the pattern similarity with Galois theory.)

## – Proemial Relation according to Gotthard Günther

Through the billowing up of the number line, the novel numbers such as $\sqrt{2}$ or $\pi$ can indeed be admitted into the set of numbers extended from the natural numbers $\mathbb{N}$ to the real numbers $\mathbb{R}$, but there must be at least one point at which the billowing stands perpendicular to the linear number line. This point can theoretically lie anywhere on the number line, but in the sense of a norming simplification the zero point is chosen for it. There the imaginary axis rises perpendicularly. For its unit *i* there is no place on the number line. It stands perpendicularly above zero.

It is this step that Gotthard Günther (1900-1984) is concerned with. He leaves pure mathematics and understands the number line with its usual arithmetic (addition, subtraction, multiplication, division) as an example of a *positive language*: that is a language with rules with which one can operate on the elements of the language without thereby producing new elements that lead beyond the language; this is roughly the language formulated by Wittgenstein in the *Tractatus*. To this corresponds in mathematics the *number field*, which was introduced in 1871 by Richard Dedekind (1831-1916), after his engagement with Galois theory, in his supplements to the lectures on number theory by Dirichlet that he edited: “By a field we wish to understand every system of infinitely many real or complex numbers that is in itself so closed and complete that the addition, subtraction,

multiplication, and division of any two of these numbers always produces again a number of the same system.” (Dirichlet, 424, my translation)

Günther's positive language is comparable to the number field insofar as in both the operations lead to elements that remain within the given set. What mathematics designates as algebraically closed is for Günther the *mono-contexturality* of positive language. The language of elementary arithmetic and in particular the number field of rational numbers can therefore be seen as the paradigmatic example of positive language.

In the zero point he sees negative language arise, which stands perpendicular to positive language and withdraws from it, just as the imaginary axis with the unit *i* stands perpendicular to the number line of real numbers. In my understanding, this is a generalization of the movement into the abyss that Kunze described with the example of the city myths of Rome. This brings out the critical moment from which the movement of re-entry arises.

This separation of negative language from positive language is, for Günther, in turn the elementary step of an overarching movement that he designates as the *proemial relation*. With the proemial relation, a given level is left and a new level is entered. This can be compared with emergence and autopoiesis.

Günther goes back to the logical germ cell of this movement. For him, this is the extension of two-valued logic, resting on 'true' and 'false,' which in his view remains in positive language and is broken open by negative language. In order to understand what happens in this moment, he introduces neither a third value coming from outside nor an observer who sees all this from outside and is able to supplement a third value that goes beyond 'true' and 'false.' Rather, he investigates in the two-valued relation R(x,y) an *inner movement* that leads it beyond itself. The third value is therefore for him not the transition to a three-valued relation R(x,y,z), but an inner freedom of the three relational concepts that is already given with R(x,y). These are the two relata x and y as well as the relator R (the relation itself). He recognizes that with every two-valued logic, implicitly with the relation of its two values, a third magnitude is already given that, considered for itself, yields the third value: that is the relation R of these two values.

The limit of classical two-valued thinking lies for him therefore not simply in its two-valuedness, for with the relation of these two values it already disposes over the third value, but in its “prohibition” against relating relatum, relator, and relation to one another in turn. With classical two-valued thinking, within a relation R(x,y) only the two relata (the mutually related, the arguments x and y) can be exchanged with one another: from R(x,y) there results R(y,x). If x and R were also exchanged, a self-reference of propositions threatens that can refer negatively to

themselves (like the liar paradox ›this proposition is false‹), which Russell wished to avoid. The concern of the proemial relation is therefore to show with which formalism an exchange of R, x, and y is possible without falling into the antinomies identified by Russell. In the result it will become apparent that this can succeed only if there is an x that is itself already both operator and operand, as Spencer-Brown succeeded in doing with his mark ⌐ and as holds for the imaginary unit *i*.

This can be made clear if the relation R is understood as the subject. In classical thinking, the subject R can only exchange the objects (e.g., true and false, plus and minus, forward and backward, I and thou). What classical thinking cannot do is to place the subject R itself as an argument on the same level as x and y.

The three-valued logic sought by Günther therefore requires no external third and expressly wishes to avoid it. It accomplishes this movement as pure self-reflection – an approach that Günther developed directly out of the attempt to find a suitable formalization for Hegel's dialectic. Here is adumbrated what Spencer-Brown formalizes as operative closure: re-entry is precisely this movement running back into itself, in which the boundary becomes its own content.

At first glance it is difficult to understand how this is supposed to be possible and what Günther means by it, but he brings a vivid example (Günther 1970, 227): in nature, different orders are nested within one another. Thus an atom can be understood on the one hand as a relation in which various elementary particles such as protons, neutrons, and electrons combine with one another and are brought into an order, and on the other hand as the matter from which, in a molecule, the atoms are in turn brought into an order. This can be formalized:

$\mathbf{A} = R_1(E_1, E_2)$

read: the atom A is a relation $R_1$ of the elementary particles $E_1$ and $E_2$ from which it is composed

On a higher level, the atoms sink to the level of the relata:

$M = R_2(\mathbf{A}_1, \mathbf{A}_2)$

read: the molecule M is a relation $R_2$ of the atoms $A_1$ and $A_2$ from which it is composed

In this example, the molecule is a higher order (meta-order) relative to the atom, just as one level lower the atom is a higher order relative to the elementary particles. Essential for Günther is the insight into how in this example the arguments and relations exchange places. The atom changes from the position as a relation into the position as an element (relatum).

This change from one order into a higher or lower order is designated by Günther, somewhat obscurely, as the *proemial relation* (proemial relationship), “for it 'pre-faces' the symmetrical

exchange relation and the ordered relation and forms, as we shall see, their common basis" (Günther 1970, 226). Günther refers to prooimion (Günther 1970, 226 n. 3). This means literally 'entrance, beginning, preface, prelude,' with the root oimos ('the way, strip, a plane,' also the 'course of a song'). Günther understands by the preface or prelude the transition into a lower or higher level, which must occur before the respective order can form (emerge) within this level, from emergere, literally 'to emerge, to come forth, to rise up.'

Walter Rella proposes a different interpretation. For him, 'proemial' does not go back to 'prooimion,' but to the root emo, which stems from Latin, in the meaning 'to acquire by purchase, auction, or sale,' and refers in the literal translation 'before the purchase' to the open state when a commodity comes onto the market and must find its buyer through a price negotiation. A division-of-labour economy can be understood as a network in the three states of production, exchange, and consumption. While production and consumption each show a transitive order in which production or consumption can be carried out sequentially, the economy in the state of exchange moves in a non-transitive, circular, or virtual order in which it must show whether producers and consumers find one another and a societal equilibrium of all supplies and demands can be achieved. In this moment crises can arise, such as the cyclical crises of overproduction in capitalism or planning errors in socialist or state-organized economies. The phase of exchange is seen thus as a higher order relative to production and consumption, between which it mediates. (Email of 22 February 2020.)

## – Galois Theory

Can this be transferred from the material orders of nature with elementary particles, atoms, and molecules, and from the economy with production, consumption, and exchange, to the level of mathematics and logic? In order to be able to formalize it there independently of empirical examples, a mathematical entity x must be found that *in itself already bears the double character* of being able to act both as operand (argument) and as operator (function). Then an operative change of the form $R(x,y) \rightarrow x(R,y)$ becomes possible, in which the former argument x assumes the status of the governing relation.

This was precisely Spencer-Brown's intention when he introduced the ⌝, a symbol that possesses exactly this property. In what follows, we will show that the transition from this primary symbol to its infinite self-reference – the re-entry ⌐ – can be understood, structurally speaking, as a proemial relation in Günther's sense. This approach also avoids oversimplified interpretations that ultimately reduce the 're-entry' to classical, two-valued Boolean logic, as already done in the critique by Cull and Frank, who polemically speak of *Flaws of Form* rather than *Laws of Form* and view Spencer-

Brown's work as merely a rehash of George Boole's logic. Regrettably, Rudolf Kaehr, Günther's most important scholar, has also followed this line of criticism (Kaehr, 43).

Mathematics knows, with the imaginary unit *i*, such an entity, upon which the sought double character of relatum and relation can be shown formally. In what follows it is to be shown how a proemial relation can be constructed with *i*.

To begin with, *i* is a newly introduced number that solves the equation $x^2 = -1$, which is unsolvable in the real numbers, and geometrically stands above the zero point of the real number line. In order to admit it, a field extension is required: the ground field $\mathbb{R}$ is adjoined with the element *i* to form the new field of complex numbers: $\mathbb{C} = \mathbb{R}(i)$. Here *i* occurs purely as a *static* operand J.

$$J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \text{ mit } J^2 = -I$$

For the condition $J^2 = -I$, I denotes the 2×2 identity matrix (not to be confused with the imaginary unit *i*).

When in what follows the mathematical structure of the proemial relation is unfolded, it is to be shown and used that *i* is not only a number but at the same time an operation. Philosophically speaking, what is at stake is the fundamental double meaning of being and function.

In order to find a formal generalization for this constellation, one must start from a ground field (such as the field $\mathbb{Q}$ of rational numbers). If unsolvable equations occur there, this leads to new elements that must be added as solutions and that lie outside the given field. Such a new element shall be designated generally by $\alpha$. In order to integrate $\alpha$ mathematically, a new field must be constructed with which the ground field is extended by these elements, which is called adjunction. In this extended field the familiar laws of calculation of addition and multiplication continue to hold unchanged.

For every such extension a so-called splitting field can be constructed: this is the smallest possible extension field that contains not only $\alpha$ but all sibling solutions symmetric to $\alpha$ (the algebraic conjugates). For this splitting field the corresponding Galois group G is now determined. It describes the inner symmetry relations between the new solutions.

The classical example was one of the greatest crises in the history of mathematics: for the equation $x^2 = 2$ (which is geometrically the diagonal in a square with side length 1) there is no rational number – that is, no ratio a : b of two integers a and b – that comes into question as a solution. This discovery plunged Pythagorean mathematics into a fundamental crisis of foundations, which had

assumed that the world is built up out of natural numbers and their ratios. This philosophical (and, among the Pythagoreans, religiously motivated) basic assumption had to be abandoned. For the solution, the two new numbers $\sqrt{2}$ and $-\sqrt{2}$ must be introduced. They lead to the extension field $\mathbb{Q}(\sqrt{2})$. The novel radical sign $\sqrt{\ }$ was used only from the sixteenth century onward.

These two solutions stand in a clear symmetry to one another: they lie mirror-image on the number line. Galois theory generalizes this intuition: it asks which structural symmetries (e.g., through exchanges or mirrorings) exist for the novel solutions *α*. This step, carried out by Galois, led from the mere calculation with numbers to the investigation of symmetry structures and algebraic *paths of solution*: he no longer simply asked with which numbers given tasks can be solved, or which numbers must be newly constructed in order to be able to solve them, but he asked according to which rules the field extensions proceed step by step. He investigated, in the paths of solution of the stepwise extensions, their properties, and was thereby able to clarify questions that had remained unanswered for millennia, such as whether solutions *can* exist for the trisection of the angle or the squaring of the circle within Euclidean geometry. He was able to demonstrate that this is not possible, and that it is therefore pointless to search further for such solutions. Instead, as in this example, extensions from a Euclidean to a non-Euclidean geometry are required. His theory was, forty years later, the stimulus for Dedekind to introduce the concept of the field into mathematics. It will become apparent that with this method a path of solution can also be found for self-referential propositions without falling into Russell's antinomy.

Proceeding from this, the superordinate relation R(*α*, G) can be considered, which assigns to the adjoined element *α* its corresponding Galois group G. It is now the aim to show that this linkage possesses the structure of a proemial relation in which the algebraic object *α* and its symmetry structure G can exchange their functional places. With this relation the concrete element *α* is directly coupled with the structure of its own automorphism group.

This shall be explained with the concrete case of the field extension from the real numbers $\mathbb{R}$ to the complex numbers $\mathbb{C}$. The corresponding Galois group G = Gal($\mathbb{C}$/$\mathbb{R}$) consists of all automorphisms of $\mathbb{C}$ – that is, the structure-preserving, bijective mappings of the field onto itself – which leave the elements of the ground field $\mathbb{R}$ invariant (unchanged). It turns out that exactly two such mappings exist: on the one hand the trivial identity (*id*), which maps every complex number onto itself, and on the other hand complex conjugation (σ), which flips the complex number plane mirror-image about the real axis. It assigns to every complex number above the real number line its mirror image below the number line (and correspondingly, conversely, to the numbers below the number line the numbers above).

The group G = Gal(ℂ/ℝ) is finite (it consists only of these two elements), and it is cyclic: if complex conjugation is carried out twice in succession, the result is again the initial numbers. (This corresponds structurally to *recrossing* in Spencer-Brown: crossing the boundary and crossing it once more.) Within algebra, a cyclic group consisting of two elements corresponds in turn to the additive group of residues modulo 2: that is the division of the integers into even and odd numbers. For even numbers, division by 2 yields the remainder 0; for odd numbers, the remainder 1. Thus at this point the expositions of Plato on numbers, of Galois on the extension of the real to the complex numbers, and the second axiom introduced by Spencer-Brown meet. Mathematically it is represented by the group ℤ/2ℤ (the twofold division of the integers into even/odd numbers).

If this abstract group is now brought operatively into connection with the imaginary unit *i*, the so-called orbit of *i* under the action of G can be determined. This orbit is the set {i, -i}. The mathematical formalization here hits exactly what is already given geometrically to intuition: *i* and -*i* are the two mirror-image solutions of $x^2 = -1$ by which the real field must be extended.

This elaborate formal conceptual apparatus is chosen in order to generalize the geometrical intuition of imaginary numbers and to be able to transfer it as a structural principle to arbitrary extension elements *α*. As an intermediate result it can be recorded: the relation R(*i*, ℤ/2ℤ) is mathematically strictly well-defined. It mediates between a concrete algebraic object (the number *i*) and the abstract symmetry group (the additive group ℤ/2ℤ of residues modulo 2) that permutes its conjugates. Thus it becomes the paradigmatic example for the sought relation R(*α*, G).

Thereby everything is prepared in order to be able to carry out the actual *magic trick* of Gotthard Günther: the radical exchange of places, in which a thing *i* suddenly becomes alive and acts as operation (mapping/relation), which mathematically leads into modern category theory.

If now, in this well-defined relation R(i, ℤ/2ℤ), the element *i* is to be exchanged with the relation R, the imaginary unit *i* must flip from a rigid object into an active relation. Mathematically this is entirely legitimate: *i* is no longer considered as a number, but as the general *prescription* ›z ↦ iz‹, with which any complex number z is mapped onto the product ›*i* times z‹. This prescription is, on the complex number plane, the operator that takes every number z and rotates it by 90°. Expressed as a mathematical relation, this corresponds to the graph of this multiplication, that is, the set of all pairs {(z, iz) : z ∈ ℂ}.

What happens here goes back in turn to a basic idea of Dedekind. Fields of numbers become fields of functions, and because both fields are identical in their algebraic structure, they can be mapped onto one another from a more abstract view and exchanged as in the proemial relation. In 1882, together with Heinrich Weber, in the work *Theorie der algebraischen Functionen einer*

*Veränderlichen* [Theory of Algebraic Functions of One Variable], he considered functions as objects with which one can operate as with numbers. Until that point, functions were mostly understood as assignment prescriptions f(x) that assign to every x its function value f(x). Dedekind and Weber transferred Dedekind's previously developed concepts of arithmetic number fields (such as the rational or real numbers) to analysis and geometry. They showed that the totality of certain functions (concretely: algebraic functions on a Riemann surface) satisfies under addition, subtraction, multiplication, and division the same structural laws as numbers. They thus form a field, more precisely an algebraic function field. This could in the twentieth century be generalized once more to category theory, and thereby gave the framework within which Galois theory can be understood as a proemial relation.

Dedekind's discovery of treating functions like numbers was the historical spark; category theory is the architectural elaboration; and Günther's proemial relation provides the philosophical conceptual network for it.

In everyday life this corresponds to a fundamental change of perspective: a mere thing (the thing *i*) becomes an action (rotation in space). The new, superordinate relation R now no longer investigates isolated numbers, but how these actions interact with one another. A relation between relations arises – a meta-relation of the second level.

This is precisely the movement that Günther describes with the proemial relation: the relatum becomes the relation. In modern category theory this change of the level of consideration is radically generalized. The world of static sets is left, and a change to the type level occurs: from static objects to dynamic morphisms (arrows or mappings). If one goes one level higher, then these mappings are in turn considered as objects, between which new mediations – so-called *natural transformations* (2-morphisms) – operate.

This deeply self-referential step, in which an operator becomes its own argument, encountered in classical logic (as in Bertrand Russell's type theory) strict prohibitions in order to avoid paradoxes. Modern mathematics has, however, found convincing solutions for this. In category theory such a re-entry can be formalized as a reflexive object that is defined via the isomorphism ($A \cong A \to A$): here the situation is considered when an equivalence is given between the set of “things” A and the relations $A \to A$ of these things to one another. This means: the object A is structurally identical with the space of all mappings that can operate on A itself. Every element in this system is at the same time building block (operand) and transformation (operator). Such seemingly paradoxical structures are strictly proved and established in mathematical logic (as in Dana Scott's domain theory for the semantics of programming languages). Put vividly: the system builds itself a meta-level that is large enough for an operation to be able to look back upon itself without causing a logical short circuit.

Without going further into these technical details, this isomorphism lays open the core of what the proemial relation is about. Entities with such a structural double character are required in order to accomplish the leap into a higher dimension. And with category theory a framework is required in which such entities can be considered both as elements and as operators (morphisms).

Decisive is the organic success of this leap: on the higher level of the complex numbers ℂ, a novel, emergent property shows itself that the real numbers lacked: algebraic closure (the fundamental theorem of algebra), according to which every equation finds a solution in the field of complex numbers.

And this corresponds to the concern and the birth of re-entry in George Spencer-Brown: out of the insoluble contradiction of the flat initial level of primary arithmetic and algebra, with the leap into operative self-reference, the creative order of a completely new dimension emerges.

Thereby the actual goal has succeeded: emergence, self-organization, and autopoiesis do not remain descriptive terms with which, in the style of a feuilleton, one rhapsodizes about something and fundamentally leaves open what is actually meant; rather, it is shown formally and mathematically how, out of a given initial set, a new set can arise that possesses properties that were not yet given in the initial set.

## – Conclusions and Prospects

**Russell's Antinomy**. First of all, it has not only been possible to avoid Russell's paradox, but also to specify exactly the conditions under which it can be avoided. Dana Scott (* 1932) recognized that classical set theory fails because it seeks to map all conceivable (including wild, incalculable) functions. In the reality of logic and computer science, however, only those functions are of interest that can be calculated step by step through successive approximation. Category theory has created the framework for this. With the proemial relation it can be shown, within this framework, how such a stepwise construction is possible and leads on the respectively new levels to results that are recognizable and demonstrable only there and not yet on the preceding levels.

**Hegel's Double Negation**. Russell was originally a Hegelian, but came to the conviction that it is impossible to find a formal presentation for Hegel's double negation. Günther took this up anew. Looking back at Hegel, the proemial relation shows the conditions to which a successful double negation is bound. It can be applied only to concepts that themselves already contain, in their interior, comparably to the imaginary numbers and to the mark ⌐, a double meaning of operand and operator, which can then be unfolded until it leads into a new immediacy. Precisely such concepts has Hegel chosen when, for example, by *beginning* [*Anfang*] both the point at which a beginning is made and the process of beginning are meant. Likewise, concepts such as

'determination' [*Bestimmung*] designate both the process of determining and the result of this process.

**Spencer-Brown's Modulator Function**. Rudolf Kaehr reproached Spencer-Brown with the charge that, with his construction of re-entry as an infinite echelon, he falls back into countable infinity (which Hegel designates as “bad infinity”) and therefore remains stuck in a flat, monocontextural series (Kaehr, 43). When later engineers and commentators make of the modulator function sketched by Spencer-Brown a mere flip-flop circuit (thus Oksas 2021), this is admittedly a technical masterstroke, but on the whole they confirm in their way Kaehr's critique and degrade metaphysical re-entry to a simple technical component of classical, two-valued computer logic.

With the proemial relation, by contrast, the way opens for a more far-reaching understanding: the modulator can then be conceived as a continuation of that oscillation in which the double character of the imaginary magnitude *i* shows itself. The oscillatory movement is continued from the understanding of mere oscillating numbers to the understanding of paths of solution that initially find themselves in an impenetrable labyrinth and, thanks to their oscillatory movement, can reach a higher space. The orientation signs attached by Spencer-Brown to the markers of the modulator function (LoF, 67) function therein as local signposts. They are more than just static control variables. Spencer-Brown attached and entered them in his diagram still from outside. In quantum field theory they are understood as the local field strengths given by nature, which become operators in order to describe the dynamics of electron orbits. The electron recognizes, as it were, the respectively local field strength as the orientation sign indicating where it is to move next, and mathematics describes how it operates with the local field strength in order to continue its path. Category theory and the theory of fiber bundles in turn offer the mathematical framework for describing such a new understanding formally and exactly. This conceptual transition is possible if it is conceived as a proemial relation from the static orientation signs to local operators. The modulator function then no longer shows a machine that, like an electronic circuit, is calibrated to a fixed sequence, but becomes an image in which the dynamic process becomes intelligible, whose orientation signs are given in the thing itself and are not merely recognized and used as orientation signs, but have become the operators that actively carry the process forward.

Matter and the medium in which the modulator function runs are then no longer merely disturbing variables that have to be taken into account and compensated for. Rather, their cosmic pattern (*li*) is recognized in them. This pattern is not decomposed and overwritten by an external order, but is, as it were, laid bare and allowed to be granted and unfold in its degrees of freedom, as the Chinese philosopher Zhuangzi (c. 350-290 BCE) described it in his famous example of Cook Ding. The

modulator function then becomes the most convincing example of how the *Laws of Form* open the way to the Daoist thinking of which the motto chosen by Spencer-Brown speaks.

**Economy**. Luhmann, with classical economics, proceeds from an ideal economy of equal business partners in which equilibrium processes occur as in classical mechanics. In every payment, values are exchanged in the same way (money for goods or services) as impulses are in the collision of two mechanical particles. With Marx it may be doubted whether these conditions are given, but the question here is whether and how a proemial relation can arise out of such a system.

If crises occur, a chain reaction threatens: if a buyer cannot pay for his goods, the seller in turn lacks the money to be able to buy goods himself, and so on. The self-reference of the medium of money has a destructive effect. In such a situation Luhmann, like the monetarists, proposes to make money cheaper through a skilful interest-rate policy, so that things can continue. That is not yet a proemial relation, but keeps the flip-flop-like cycle fluid.

But Luhmann has prepared how a proemial relation can come about: unlike Marx, for him the germ cell of the economy is not the commodity with its double character of use value and exchange value, but the respective payments as “temporary, punctual events” (Luhmann 1988/1994, 17). With time, in accordance with the process of field extensions in Galois and with the imaginary axis, the dimension is opened along which the proemial relation can come about: money changes its type level. It is no longer merely a static building block (operand in the exchange of goods), but with *money creation* [*Geldschöpfung*] it becomes an operation (operator/steering of the payment transaction). With money creation a new, imaginary element $\alpha$ is adjoined, for example central bank money, derivatives, or options on future (not yet existing) growth. Where Luhmann's classical economics still fled into the semantics of scarcity in the case of systemic crises, the modern system consequently does not collapse in the infinite flip-flop; it uses the crisis of the first level rather as paradoxical fuel for a proemial “field extension.”

Re-entry thus does not lead here to paralysis, but to the birth of a meta-level (emergence). The system does not close itself blindly in upon itself; it builds up a universe in which the crisis of the first level becomes the productive building block of the second level.

That this step is not represented by Günther's proemial relation in the systems theory of Luhmann and his school could be due not only to the fact that the expression 'proemial relation,' unlike 're-entry,' sounds unwieldy and logically intricate, but beyond that to the fact that Kaehr (the scholar and successor of Günther), as already shown, interprets the *Laws of Form* and with them re-entry misleadingly as a mere monocontextural flip-flop circuit and reproached Spencer-Brown with remaining within Boolean thinking.

To what extent more recent systems-theoretical designs – for instance by Dirk Baecker on second-order observation in the banking system or by Elena Esposito on the temporal doubling of money in derivatives markets – carry this proemial relation in substance without explicitly designating it as a proemial relation into a polycontexturality would have to be considered in detail within the framework of a further investigation. For the moment the hint suffices that only the mathematical-logical perspective of field extension provides the formal foundation for this evolutionary rescue of the system.

Should – as in reality – this too fail and money creation rather become an accelerant of new crises (on this Baecker 2008 in *Warum kam es zur Finanzkrise? Zweimal null ist eins* [Why Did the Financial Crisis Occur? Two Times Zero Is One]), one would have to go back once more to the foundations of the given economy and possibly conceive a proemial relation of another kind that leads from the competitive economy of a distorted-competition initial situation, through the capacity for cooperation of the market participants, to a common-good economy (*common wealth*).

**Religion**. In Luhmann's systems theory, religion draws the fundamental distinction between immanence and transcendence (this world and the other world) into the system itself (thus in his posthumously published book *A Systems Theory of Religion* [The Religion of Society]).

The reductive simplification: if this religious re-entry is reduced monocontexturally, this leads to a rigid, dogmatic truth (fundamentalism) or to an empty-running ritualism. Transcendence is then seen like a this-worldly object: God becomes a *thing* within the world. The system oscillates blindly and incapable of genuine transcendence.

The proemial opening: if re-entry is understood via the proemial relation (the field extension), then transcendence becomes the “imaginary space” *i* of religious conceptions and imaginings, which stands perpendicular to immanence. Religion does not calculate along in the world, but mediates – like the Galois group – via the symmetries of the transitions of this-worldly existence into the transcendent horizon of meaning. How these symmetries are to be determined more closely on the *path of solution* from immanence into transcendence would be the task of a further investigation.

However, Luhmann and Fuchs give first important hints in *Reden und Schweigen* [Speaking and Silence]. The conversation with the transcendent can occur only in an oscillation of the sayable and the unsayable, of concealing and revealing. Luhmann and Fuchs are convinced that these are temporal processes, as in the economy. Seen thus, what is at issue are symmetries that emerge and become recognizable in temporal courses. The human soul encompasses more than the conscious acts of thought and is moved in religious experiences into unfathomable depths that withdraw from modern science, which remains within binary thinking. “This topical quality intertwines with the

temporal dimension for the purpose of dissolving (or better: invisibilizing) the paradox.” (Luhmann and Fuchs, 192, my translation) From the perspective of, and under the pressure of, the science that predominates today, this is a deficiency, and this science misunderstands the oscillation of concealing and revealing as a defensive act of veiling behind which religion wishes to make itself invisible (invisibilize itself) before the critique of science. From its own perspective, what is at issue is an overarching movement that moves the soul in its entirety and thereby leads to a proemial movement into the domain of the transcendent.

The novel proemial relation previously addressed for the economy could be supported by the social doctrine of the Catholic Church and its fundamental values of personality, subsidiarity, solidarity, and common good. The proemial relation then leads from the level of individual believers, who each wish to bring immanence and transcendence into equilibrium for themselves, through their trust in God to the community (the experience of Pentecost).

**Transition to Abjection (Kristeva)**. The difference between purely engineering thinking and metaphysical thinking is not simply an academic trifle, but can have existential effects and tear open inner abysses. The abysmalness (*diastema*) seen by Plato can show itself as a complete collapse of a human being's existence. This is described by the French philosopher and psychoanalyst Julia Kristeva (* 1941) in her works on abjection. Abjection shows what can happen when this categorial change of dimension radically fails.

The re-entry demands that the subject (the form) take the boundary into itself and be able to emerge into a meta-level in which it finds its room for movement (emergence/constitution of the I).

If this leap onto the higher level is missed, the boundary between inside and outside, between operator and operand, collapses. That is Kristeva's state of the abject. This abject (such as the corpse, the refuse, the absolutely foreign) is neither subject nor object. It is that which withdraws from order, which cannot be “adjoined” and threatens to blow the system apart from within.

Mathematically speaking: it is the paradox that has not been transferred by a field extension into a higher symmetry, but remains as an unresolved contradiction within the system and there exercises psychic and logical terror. Abjection is the horror of the unresolved paradox, the reverse side of re-entry via the proemial relation and of its creative redemption.

# Abjection as Failed Re-entry (Kristeva)

The canonization identified by Kunze and the negative language conceived by Günther have a dark side: complete failure, leading all the way to annihilation. This is far more radical than a sophistical fallacy, an attachment (in the Buddhist sense of clinging) to the superficial, or a negation that can be

successfully negated once again. It is the failure of re-entry, which, through gradual radicalization, leads to abjection.

How can this come about? Hegel assumed that with him philosophy had been completed and that the further history of philosophy had come to an end. What danger is bound up with negative self-reference and the negation of negation shows itself, contrary to his expectations, in art and in various clinical pictures that have been investigated since the 1970s by a new generation of psychoanalysis. In 1980 Julia Kristeva published her essay on abjection under the title *Pouvoirs de l'horreur* (*Powers of Horror*). The Latin *abiectus* originally means 'carelessly thrown down, pressed to the ground, low, mean, faint-hearted, despondent, contemptible, cast off.' In French, *l'abjection* sounds far more drastic and can be translated as 'abjectness, deepest humiliation, baseness, shamefulness, disgrace, infamous deed'; see e.g. dict.leo.org. For Kristeva, abjection describes a psychic constitution in which subject and object, inside and outside, cannot be clearly separated and set over against one another, and with the abject the monstrous phenomenon by which subjects feel seized and from which they cannot detach themselves. She names as examples panic attacks, phobias, borderline personality disorders, eating disorders, depressions. In an almost literary language she writes, using the example of eating disorders:

> “Loathing an item of food, a piece of filth, waste, or dung. The spasms and vomiting that protect me. The repugnance, the retching that thrusts me to the side and turns me away from defilement, sewage, and muck. The shame of compromise, of being in the middle of treachery. The fascinated start that leads me toward and separates me from them. Food loathing is perhaps the most elementary and most archaic form of abjection.” (Kristeva, 2)

Objects vital to survival, such as food or nourishment, are felt to be a threat. The summons to eat appears as a betrayal of one's own higher insight into the true, to-be-rejected character of things. People with eating disorders fall, in their behavior, into a negative self-reference that in many cases leads to death. The subject feels at once driven from outside and nevertheless proud of its supposed achievement. Objects can no longer be experienced and judged objectively, nor can the subject itself be experienced in a subjectivity secure of itself. From the perspective of the outsider, all accustomed standards collapse. “The abject [is] 'radically excluded' from the space of the symbolic community 'and draws me toward the place where meaning collapses'” (Žižek, 204, quoting Kristeva, 2).

From the perspective of a psychoanalyst trained in Freud's doctrine and in Lacan, such as Kristeva, the abject is situated still below the unconscious and the drives. Every drive expresses itself in the objects that it desires or combats; the abject, however, cannot be isolated as an object for itself and perceived as an other standing over against the subject. Food is not rejected because it is negated for

whatever reasons, but it triggers loathing before any decision about acceptance or negation. What is experienced here is, in its way, without question an immediacy of experience – but without the possibility of being able to make distinctions.

The abject arises where, in the regular development of the subject, language is formed. Kristeva describes the psychoanalytic understanding of this situation as follows: when the child must learn that the mother is not always immediately available, it acquires language. It puts words in the place of the bodily experienced object. With these words it begins to describe what it desires, and learns to formulate consciously the object of its drives. The subject learns to find a new object: “Which one? Not the object of sexual drive; the mother, or her parts, or her representatives; no more than some neutral referent or other, but symbolic activity itself.” (Kristeva, 44)

The child can now feel an erotic pleasure in its own language, which takes the place of the pleasure of immediate bodily experience. In this “primordial sphere” (Husserl) everything is still intergrown: the becoming-conscious of one's own drives and their repression, temporal immediacy and deferral, subject and object, inside and outside, as well as the separation of words from the things they name. Everything is positively and negatively self-referential: the subject arises to the extent that it succeeds in separating itself from its objects. Objects, in turn, can be recognized as such to the extent that, although related to the cognizing subject, they are separated from it.

Psychic constitutions such as the examples named by Kristeva show how this process can fail. A further example is phobias: if an object (such as a dog or a horse) triggers a phobic reaction, it can no longer be separated from the overwhelming storm of feelings unleashed in the subject. In panic, the categorial order of mutually separate objects and subjects collapses.

Anyone to whom examples from psychoanalysis are unfamiliar or appear problematic can fall back on examples from art. Kristeva refers, for instance, to the horror experienced by the main figures such as Verkhovensky or Kirillov in Dostoevsky's novel *The Demons*. In more recent art history, *Abject Art* has moreover established itself as a movement of its own. Thus Cindy Sherman, for instance, counts as one of the central pioneers of the abject in art, as do Louise Bourgeois, Helen Chadwick, Paul McCarthy, Gilbert & George, Robert Gober, Carolee Schneemann, Kiki Smith, Sarah Lucas, and Jake and Dinos Chapman (cf. tate.org). Further visual examples are offered by the paintings of Francis Bacon or the films of David Cronenberg and David Lynch; in music, a work such as *I buried Paul* by Michael Gordon may be cited (YouTube). Kristeva seems to regard it as the best therapy in abjection to cautiously present to those affected corresponding examples from literature or cinema, so that in a kind of catharsis they can reduce the horror.

Art and religion touch the critical moment of the emergence of language and with it of human verbal communication. “It is within that undecidable space, logically coming before the choice of the sexual object, that the religious answer to abjection breaks in: *defilement*, *taboo*, or *sin*.” (Kristeva, 48). Art and religion describe the phenomena of abjection. They can, however, be no more objective toward them than is possible for those affected by abjection: whoever suffers the fate of abjection counts to his surroundings as defiled and finds himself in the state of shame and sin. This is an exclusion that knows no reconciling inclusion: those affected are not only socially excluded as human beings, but because of their limited capacity to distinguish between I and thou, subject and object, and inside and outside, they are also cut off from almost every communication.

Hence the image of the devil (the serpent) in the story of the Fall is highly ambivalent: it can embody the power of pure negativity, but also the to-be-excluded danger of abjection. After the forbidden eating of the fruit from the tree of knowledge, the human being can indeed distinguish between good and evil – exactly as the serpent seductively promised him. But if he approaches the question of wherein this distinction has its ground, he falls into an insoluble, negative self-reference: is the distinction into good and evil itself good or evil? In the biblical narrative, God warned against this negative self-reference; the serpent, by contrast, seduced the human being precisely to it. Was it good that the human being chose the freedom to transgress the prohibition, or was it evil? The result of this boundary transgression was that the human being became conscious of his own nakedness and shame – in the sense of the literal meaning of the abject – and began to feel shame.

How, in the face of threatening abjection, can that be put into words which makes success possible – namely to pass out of the primordial sphere into a categorial order? In this sense I understand the coinage of the catjects coined by Dirk Baecker: “Catjects are called catjects because they are neither objects nor subjects but complex categories giving rise to objects and subjects.” (Baecker 2018, 1). With the catjects, that is to be formulated positively which fails in abjection. For Baecker, George Spencer-Brown has hit, with the *Laws of Form*, precisely that level on which the catjects emerge and can be recognized in this emergence.

But how is this possible? The medium is not only the ground in which forms are entered, from which forms emerge, and which is set into oscillation in this process; rather, within this oscillation something completely new can come about, whereby abjection is avoided and the path of the catjects is opened. If the oscillation is conceived as the first emergence, then what arises from it can be designated, with Claus-Artur Scheier, as a “second emergence” that shows itself “in the moment, *systema momentaneum*, analogously to Leibniz's *mens momentanea*”:

> “On the level of basal self-reference *only this* occurs, that time generates itself as horizon (reflexivity) and immediately erases itself again as this horizon. The product is only the vanishing moment of productivity, identity only the vanishing moment of difference, *alêtheia* only the vanishing moment of *lêthê*.” (Scheier, 121, my translation)

If this succeeds, then not only is the negative self-reference of time found – which Kant indeed addressed indirectly as an idea but, lacking the concept of negative self-reference, could not yet determine more precisely (time, which immediately erases itself again as horizon in the moment of its generation) – but with it one can also speak of the *fruitfulness* of the ground, thanks to which the forms entered into it can first flourish. This leads to a second transformation of the previously described processes of re-entry: if through the first transformation the medium was set into oscillation, then from the second transformation, in the moment of oscillation, fruitfulness emerges.

While abjection consequently leads to the interruption of communication, the communication that emerges in flourishing proves to be a sign of successful linguistic capacity. With this linguistic capacity the question of the observer is inevitably raised. The observer disposes not only over a spatial distance to what happens, but also over a language with which he can record the process and put it into words; with language, in turn, the infinite multiplicity of possible interpretations is given. The standpoint of the observer results thereby not solely from the place from which he sees something and puts what is seen into language, but from his inner constitution, the *conditio humana*. This leads into a new theme that goes beyond the question of pure re-entry.

If the question is asked about the place at which abjection and linguistic capacity separate from one another, this touches on the question of chóra, discussed since Plato. Paul Natorp translates: in the indeterminate from which creation proceeds, “something positive” must lie “hidden” (Natorp, 365). This is, in a first approximation, “determinacy of place” (ibid.), “that in which it becomes,” the “receiving of the mother” (Natorp, 367), as well as the “seat” and “somehow a hold on being” (Natorp, 368) – mathematically speaking: space. Or in Plato's own words:

> “Because it is filled with powers that are neither similar nor evenly balanced, no part of it is in balance. It sways irregularly in every direction (*anômalôs pantê talantoumenên*) as it is shaken by those things, and being set in motion it in turn shakes them. (Platon *Timaeus*, 52e)

> “Indeed, it is a fact that before this took place the four kinds all lacked proportion and measure (*alogôs kai ametrôs*), and at the time the ordering of the universe was undertaken, fire, water, earth and air initially possessed certain traces of what they are now (*ichnê men echonta hautô atta*). They were indeed in the condition one would expect thoroughly god-forsaken things to be

> in. So, finding them in this natural condition, the first thing the god then did was to give them their distinctive shapes, using forms and numbers (*eidesi te kai arithmois*). ” (53a-b)

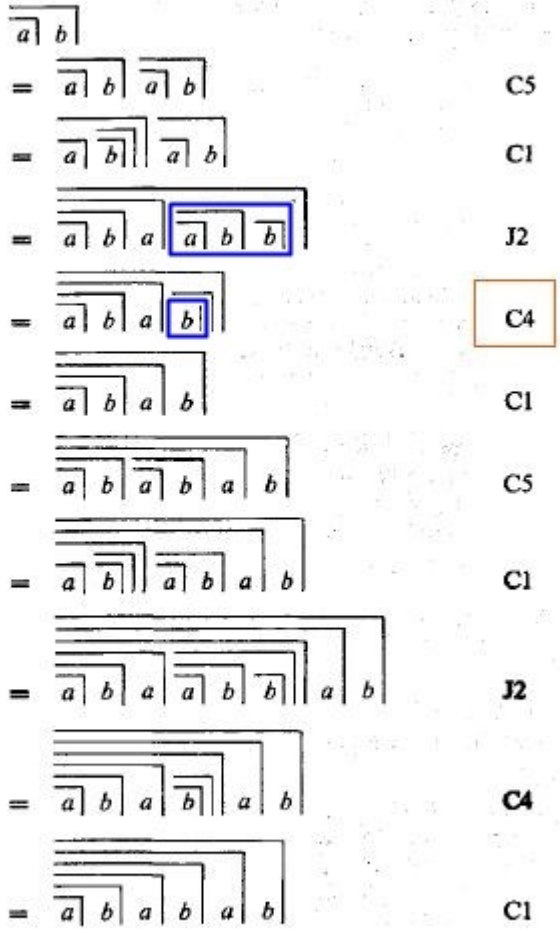
Thereby the circle closes from ancient cosmology to modern form analysis and psychoanalysis. What Plato describes as the pre-linguistic, measureless swaying (*alogôs kai ametrôs*) is precisely that primordial sphere out of which, according to Kristeva, the existential threat of abjection can arise. Only the step to formative distinction – whether through cosmic numbers, mathematical forms, or successful language – makes possible the productive re-entry. If this fails, rigidity, annihilation, and falling silent threaten. If, however, it succeeds in the sense of a second emergence, the space opens for communication, observation, and the living interplay of relations.

Decisive here is the question of fruitfulness. While the question of good and evil leads into those regions against which the Old Testament warned the human being, fruitfulness as creative potential is immediately given to the human being. It remains a genuine question of religion where this fruitfulness is situated, and when and how it becomes available to the human being. Such reflection undoubtedly leads beyond the pure formal analysis of the *Laws of Form*. George Spencer-Brown has suggested, with the motto from Lao-Tse's *Daodejing* placed at the beginning, that the *Laws of Form* are ultimately to be read as a step of enlightenment that points beyond mathematical logic. At the low point of formal and psychic collapse, it is finally religion that can give the ethical force to recognize people who suffer from abjection in their inalienable human dignity and to support them in their existential isolation.

## Appendix on Canon C9

Felix Lau offers some assistance for understanding Canon C9 (Lau, 89). He explains the rules drawn upon: C5 (iteration; LoF, 33; Lau, 79), C1 (reflection; LoF, 28; Lau, 77f.), J2 (distributive law; LoF, 28; Lau, 77), C4 (occultation, concealment; LoF, 33; Lau, 79f.). Above all, however, he concentrates on the most difficult passage in this proof:

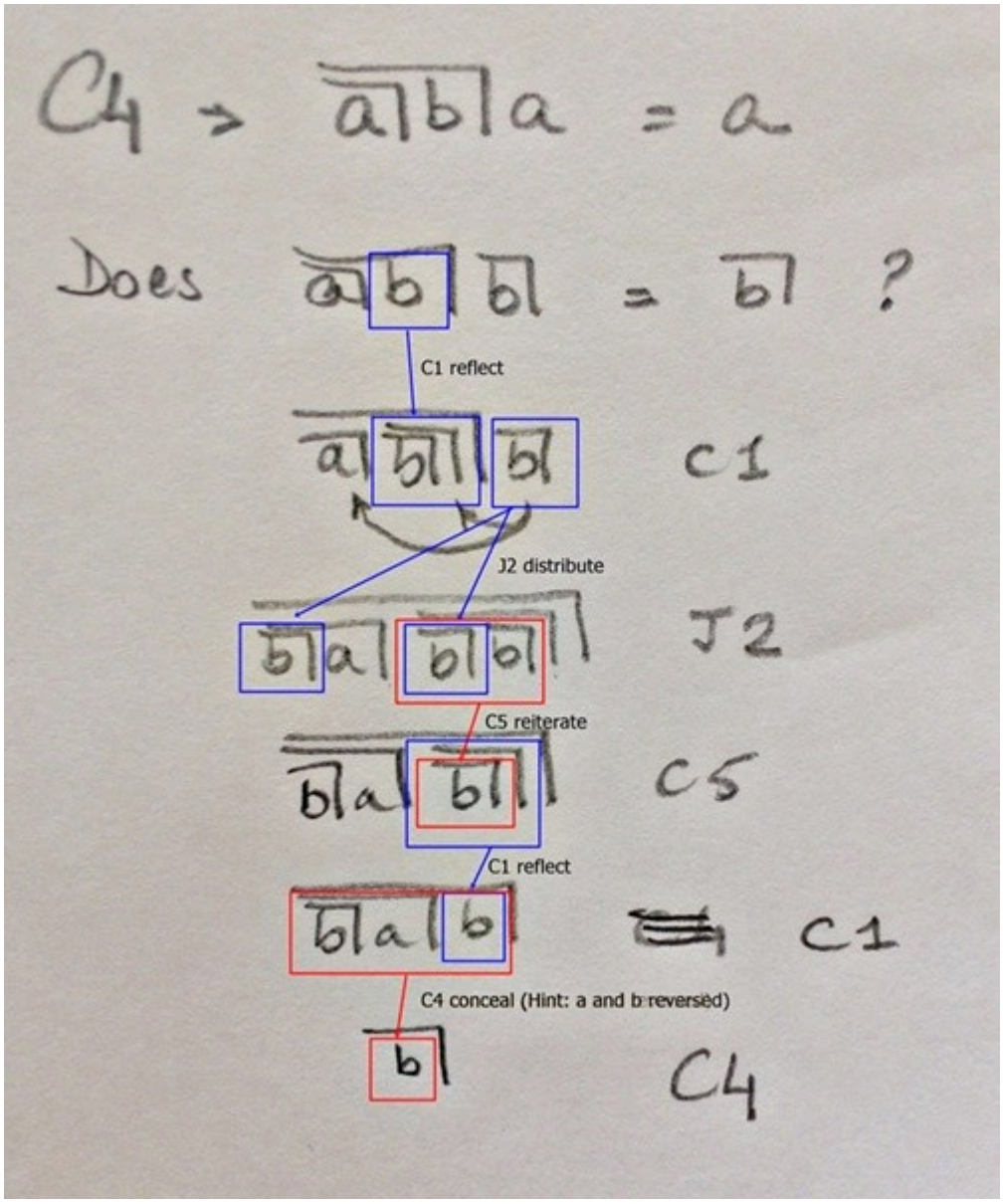
We propose now to **generate a step-sequence of the** following form.




etc. There is no limit to the possibility of continuing the sequence, and thus no limit to the size of the echelon of alternating *a*'s and *b*'s with which $\overline{a\,|\,b}$ can be equated.



The critical passage is highlighted with a blue border. Lau has shown its identity in a side calculation. He names – without elaborating further – the proof steps C1, J2, C5, C1, C4. The demonstration was achieved by Leon Conrad (email of 30 March 2020 with my comments; see also on YouTube the Laws of Form Course by Leon Conrad, Session 14):




## List of Abbreviations

AUM = The G. Spencer-Brown – AUM-Conference 1973, Esalen 1973; Link

LoF = George Spencer-Brown: Laws of Form, New York 1972 (Julian Press) [1969]; siese.org

TLP = Ludwig Wittgenstein: Tractatus logico-philosophicus
in: Werke Band 1, Frankfurt am Main 1984; Online (accessed 28 September 2026)
the numbered propositions are cited, not the page number

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